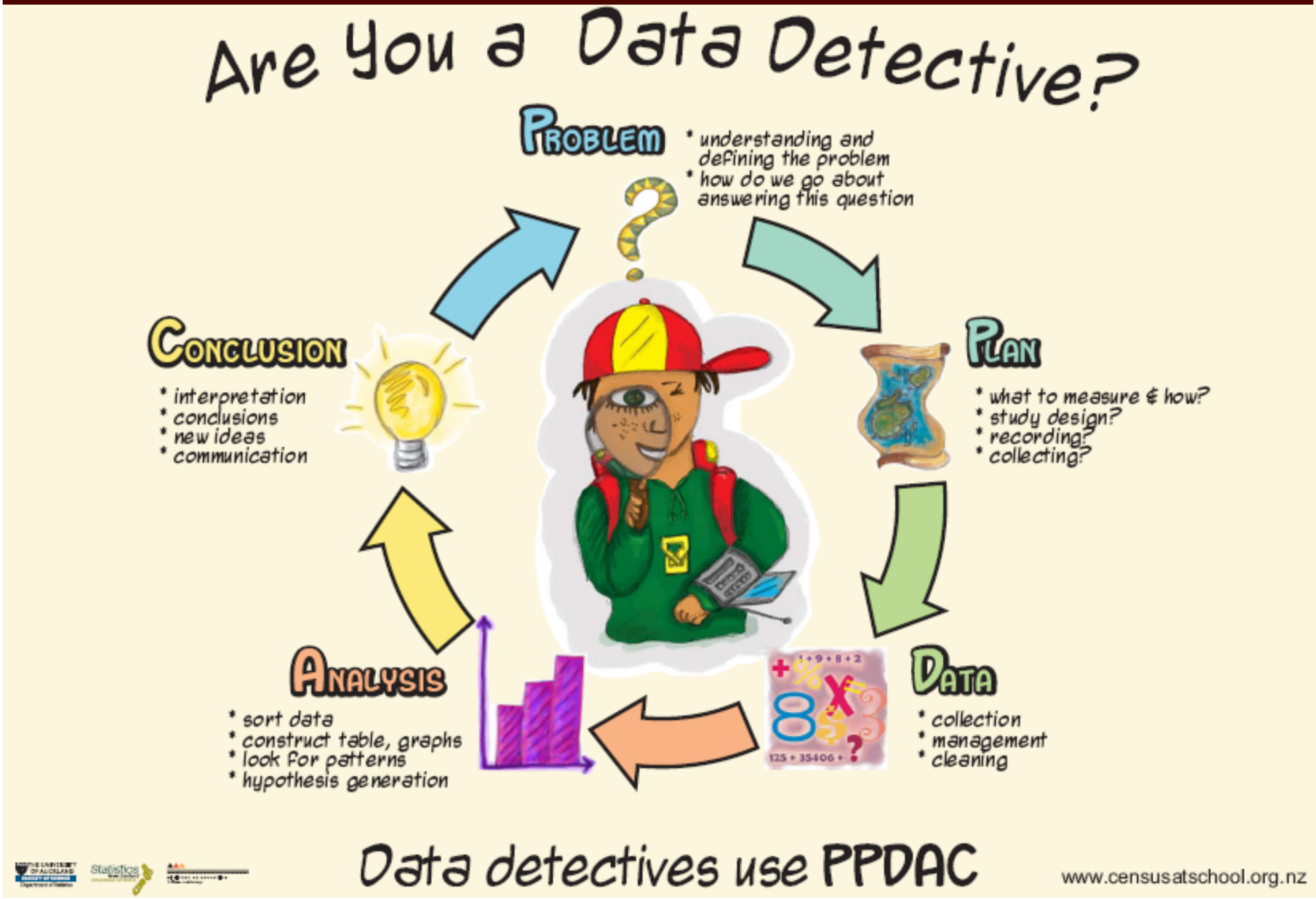
Research supported by TLRI

# Developing secondary students' inferential reasoning in statistics

Maxine Pfannkuch, Auckland University, New Zealand


NZ Teachers:
Statistics
workshops

# Statistics in the NZ curriculum



# Three strands run through the NZ curriculum (Yrs 1-13)

# Statistical inference

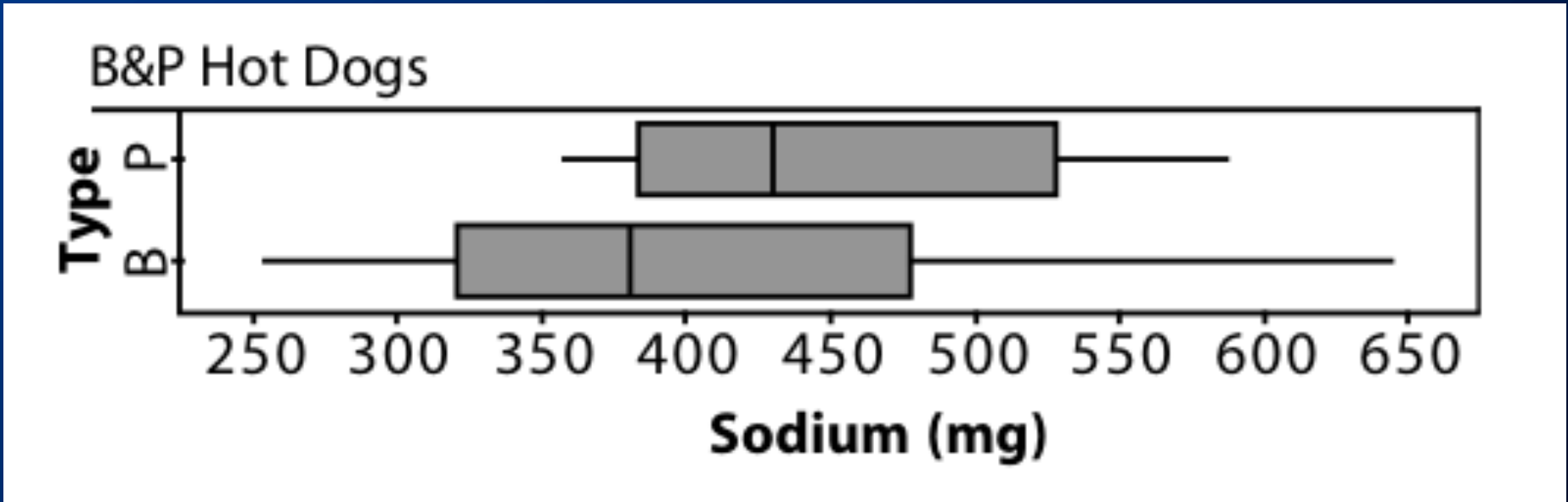

- Small part of the NZ statistical curriculum

THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# Outline of Talk

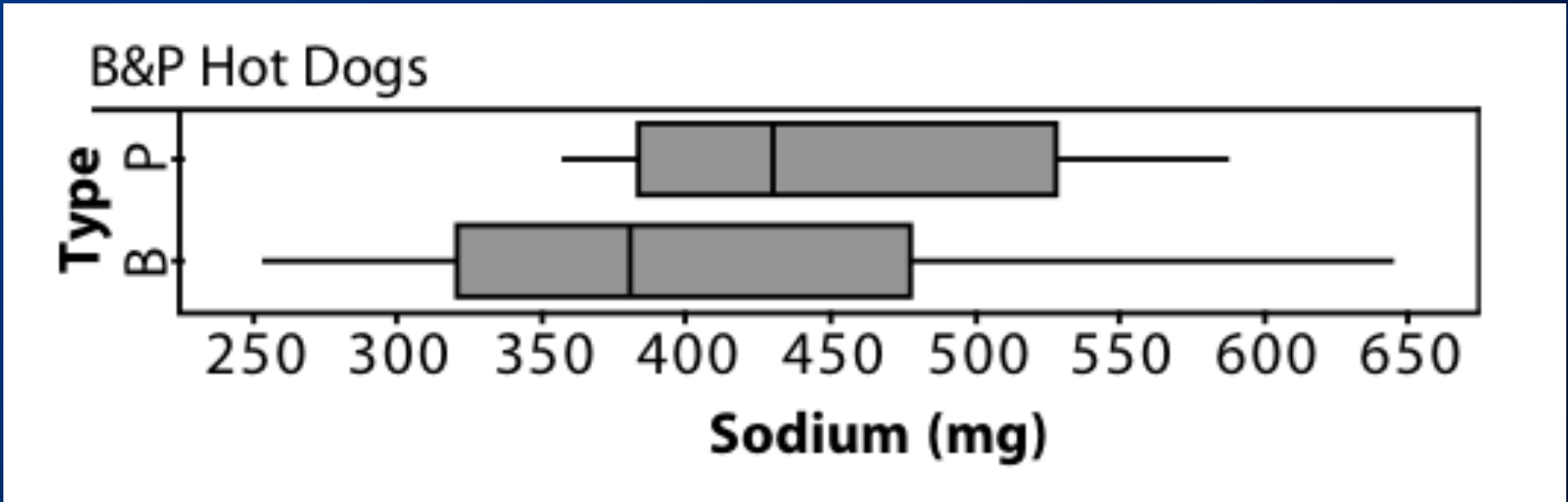
- Why inference should be in the curriculum
- Big ideas behind inference
- Our research
  - Students' initial ideas about inference
  - Sequence of instruction developed for 14 year-olds
  - Students' ideas about inference after teaching
- Conclusion

# Why should inference be taught?

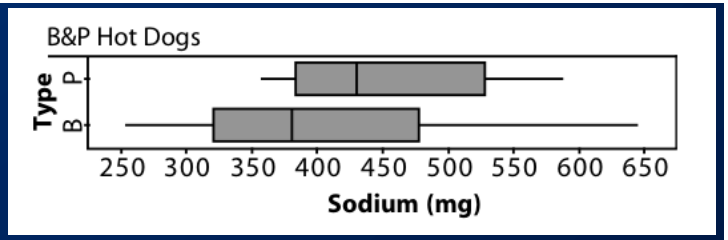



- Investigators analysed the sodium content of some beef (B) hot dogs and some poultry (P) hot dogs. This is their graph (Franklin et al., 2007).
- Discuss in pairs what statements you would make from these plots.

# Why should inference be taught?




- What statements did you want to make?
- What statements did you make?
  - Did you describe what you can see?
  - Did you infer about …?
  - Did you consider sample sizes?
    - Does it matter?




# Inference Research at School level

- Pratt & Ainley (2008) – The problem
- Students and teachers do not know whether they are playing Game 1
  - Reasoning about data as if it were the whole population
- Or Game 2
  - Reasoning about an underlying population from which the data were a sample

# Inference research at University level

- Study on students' grasp of statistical inference concepts:
  - the results from our study agree with the findings of the considerable research that has been done in the last thirty years … that people tend to think deterministically and lack awareness or understanding of variation and its relation to sample size.

(Meletiou-Mavrotheris, Lee, & Fouladi, 2007)



# Instruction at school

- Research suggests students should be introduced to inferential concepts much earlier
- Students need a deeper understanding of sampling variability (Konold & Kazak, 2008)

# Statistical inference

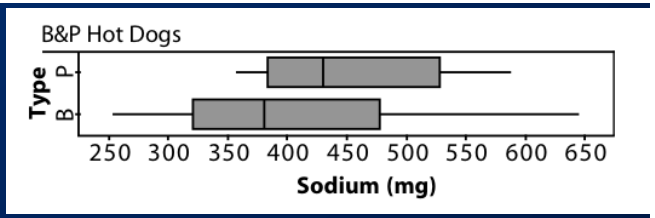
- Statistics is about

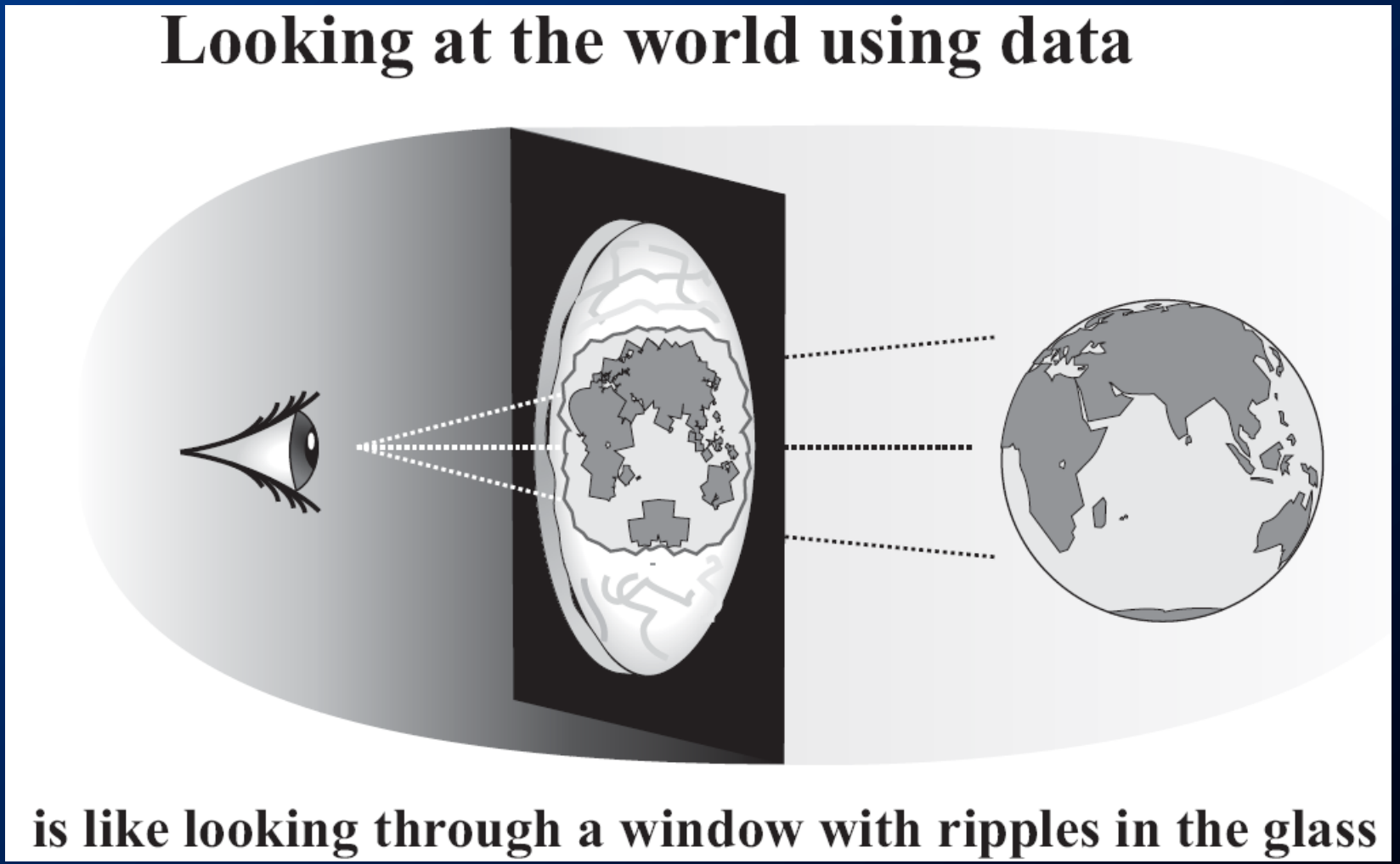
  making decisions in the face of uncertainty

- Statistical inference

  "moves beyond the data in hand to draw conclusions about some wider universe, taking into account that variation is everywhere and the conclusions are uncertain" (Moore, 2007)

# Making an inference

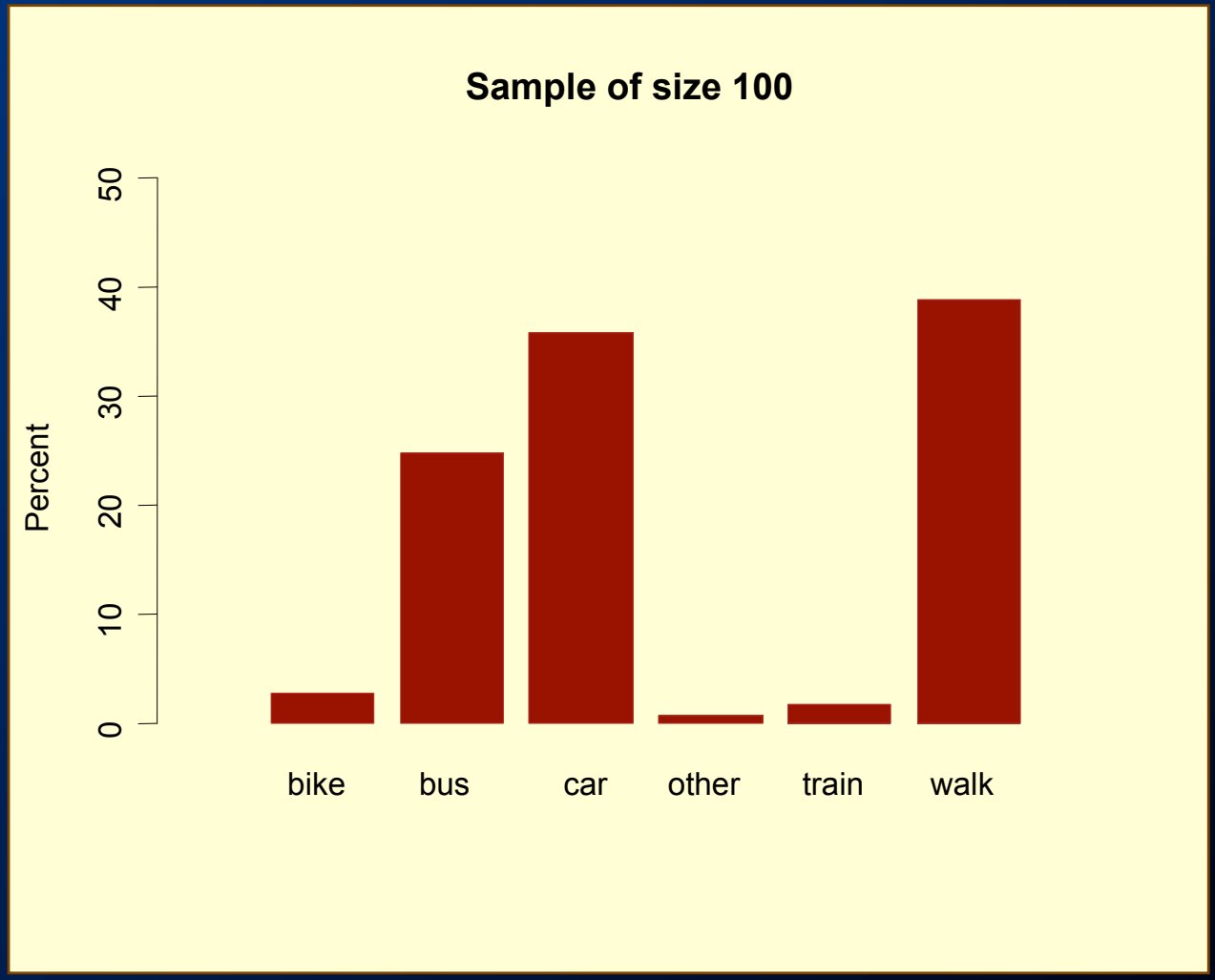


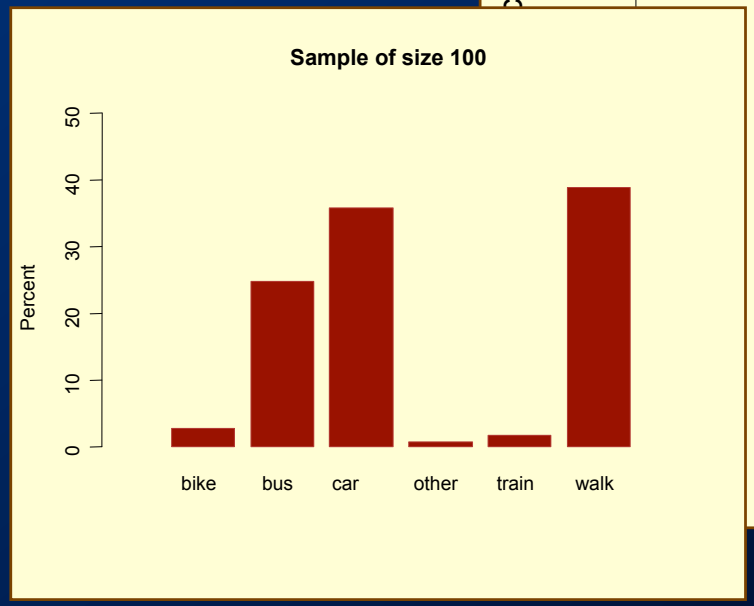

*"What I see, . . .*

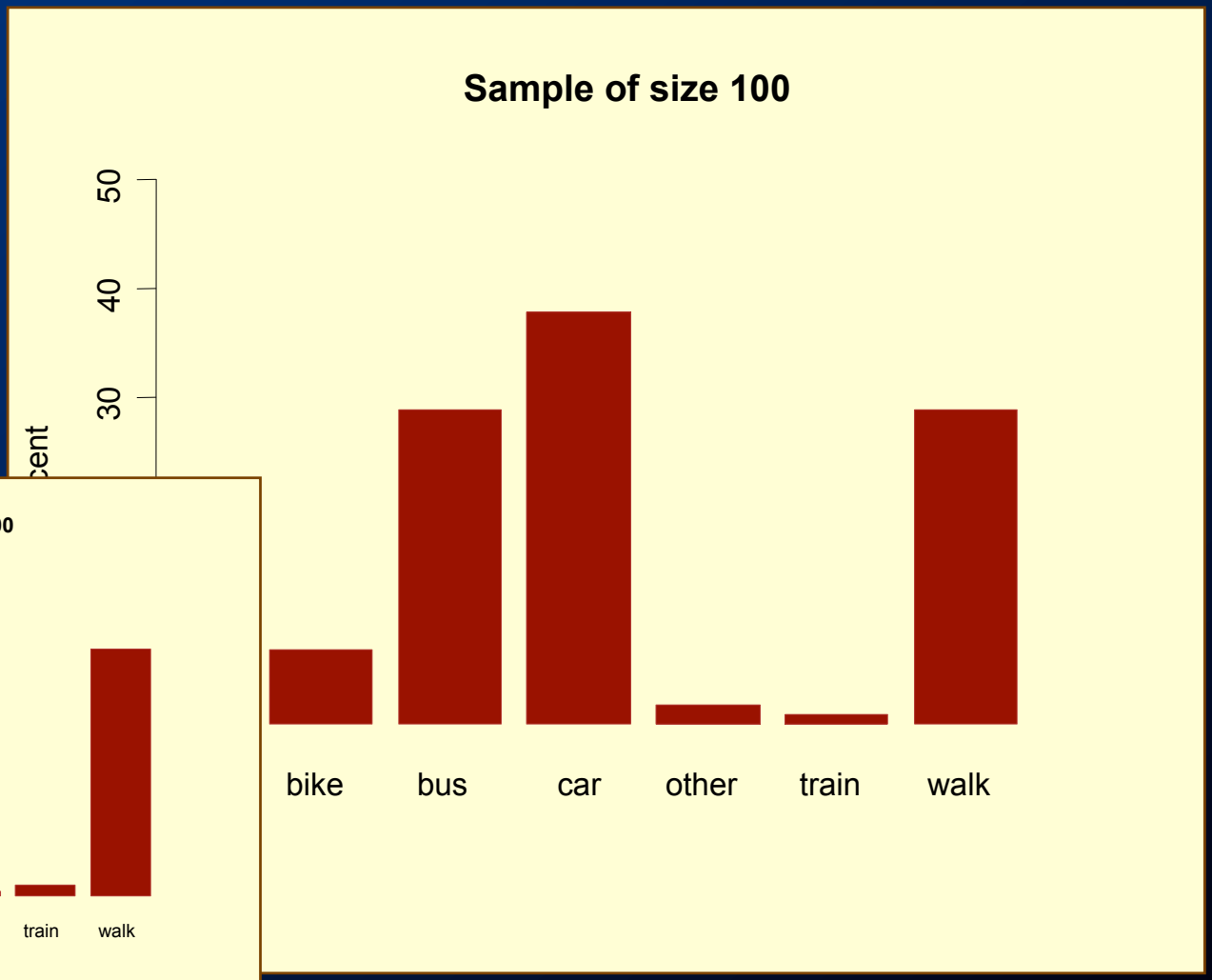
*is not quite the way it really is"*

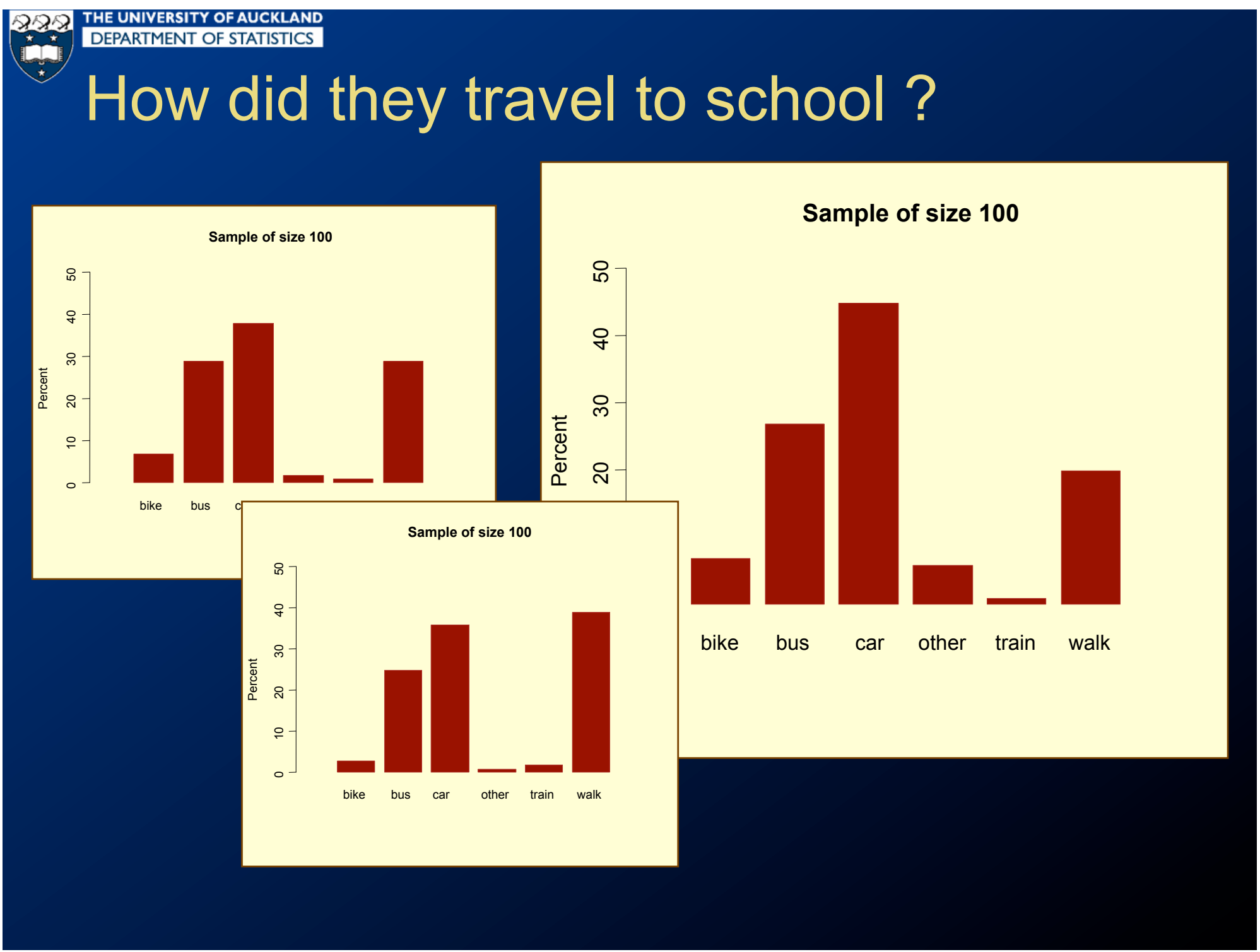
# How did they travel to school ?



# How did they travel to school ?



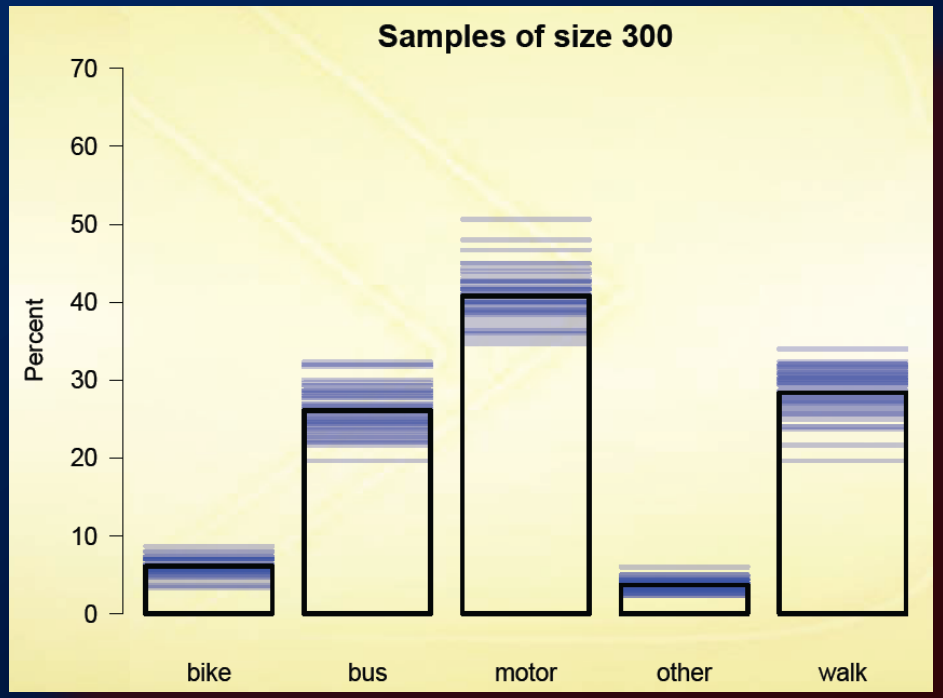




THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS
How did they travel to school ?
Sample of size 100
Percent
0
10
20
30
40
50
bike
bus
car
other
train
walk
Sample of size 100
Percent
0
10
20
30
40
50
bike
bus
car
other
train
walk
Sample of size 100
Percent
20
30
40
50
bike
bus
car
other
train
walk

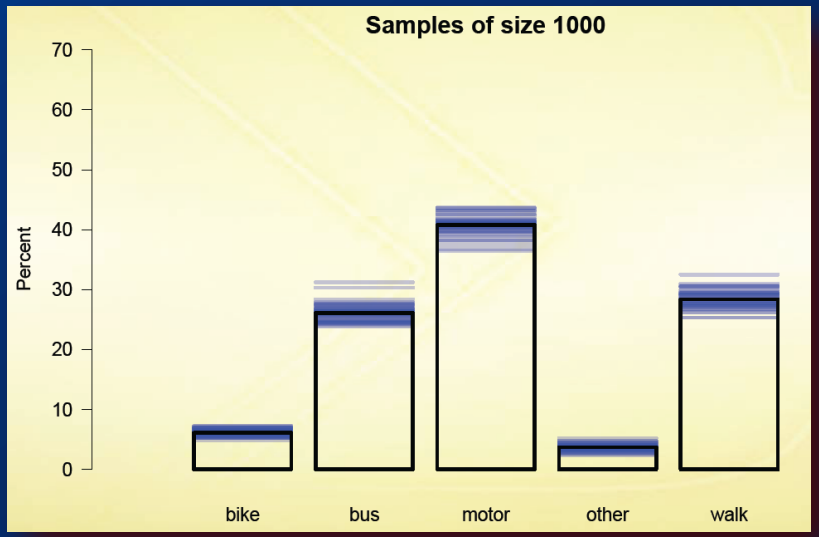
# Bar Chart Animations

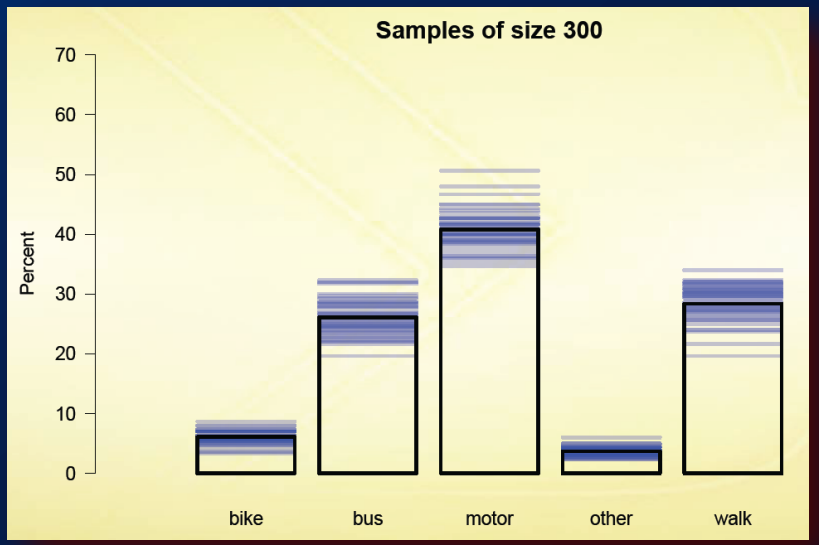
Play

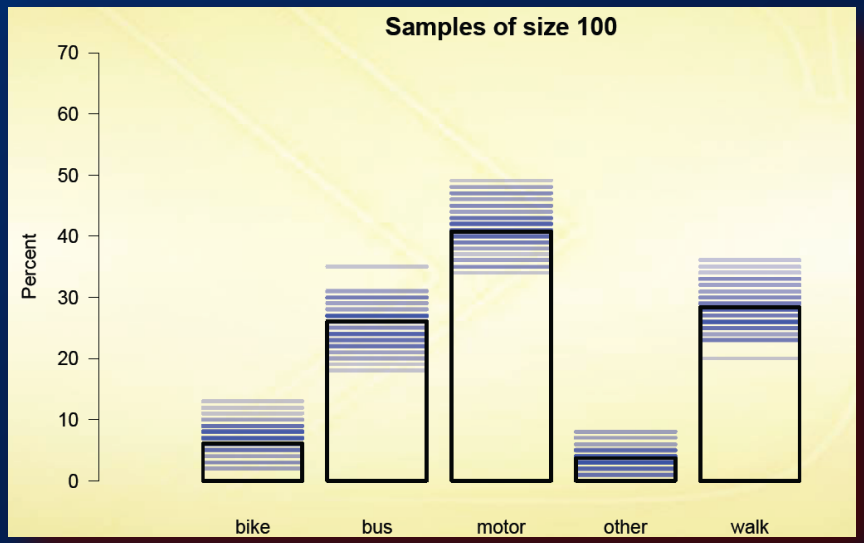
- Samples of 1000
- Samples of 300
- Samples of 100
- Samples of 30

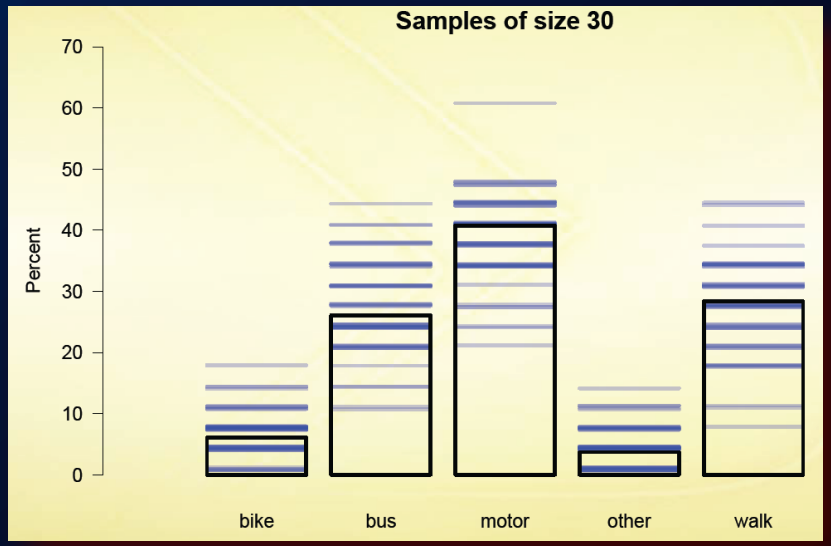


# *"What I see is not quite the way it really is"*

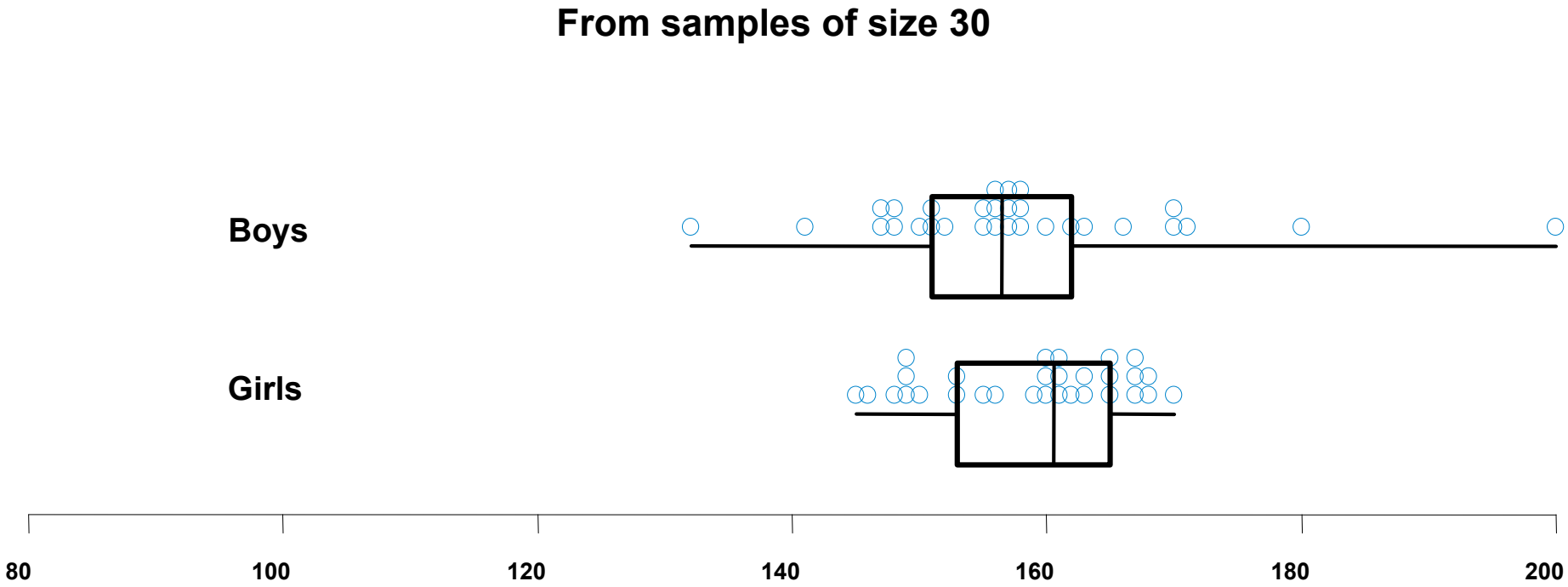


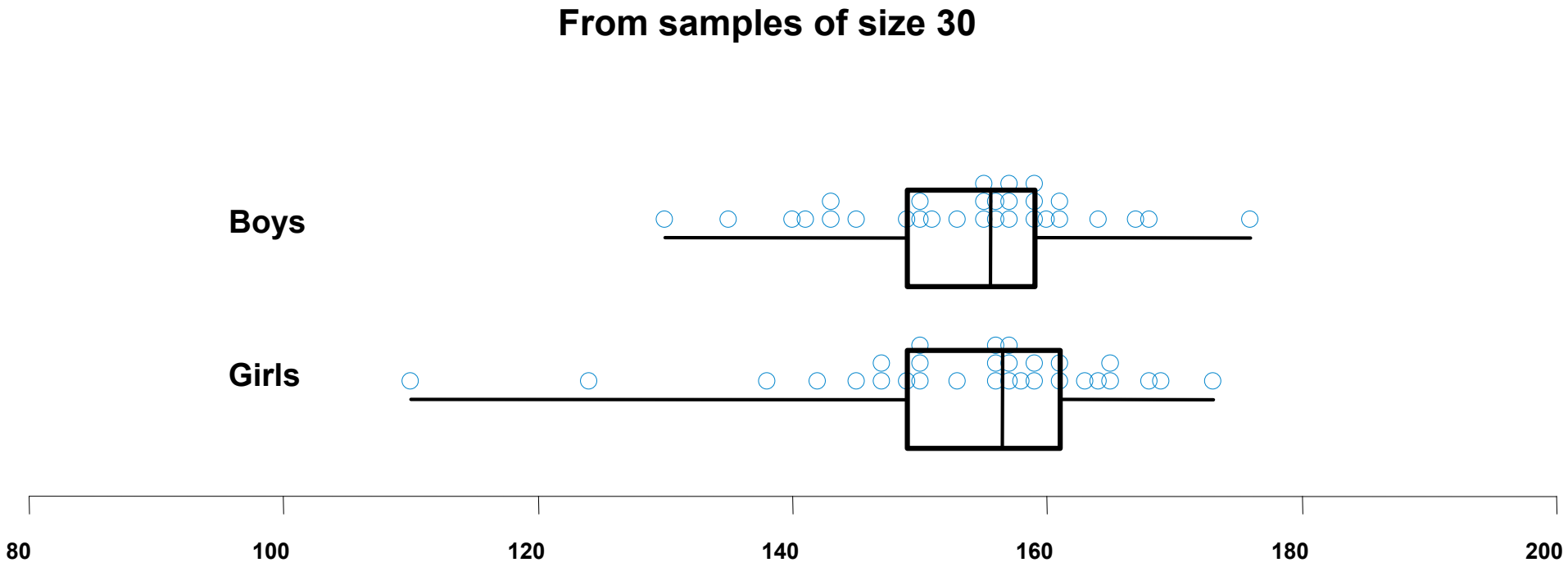



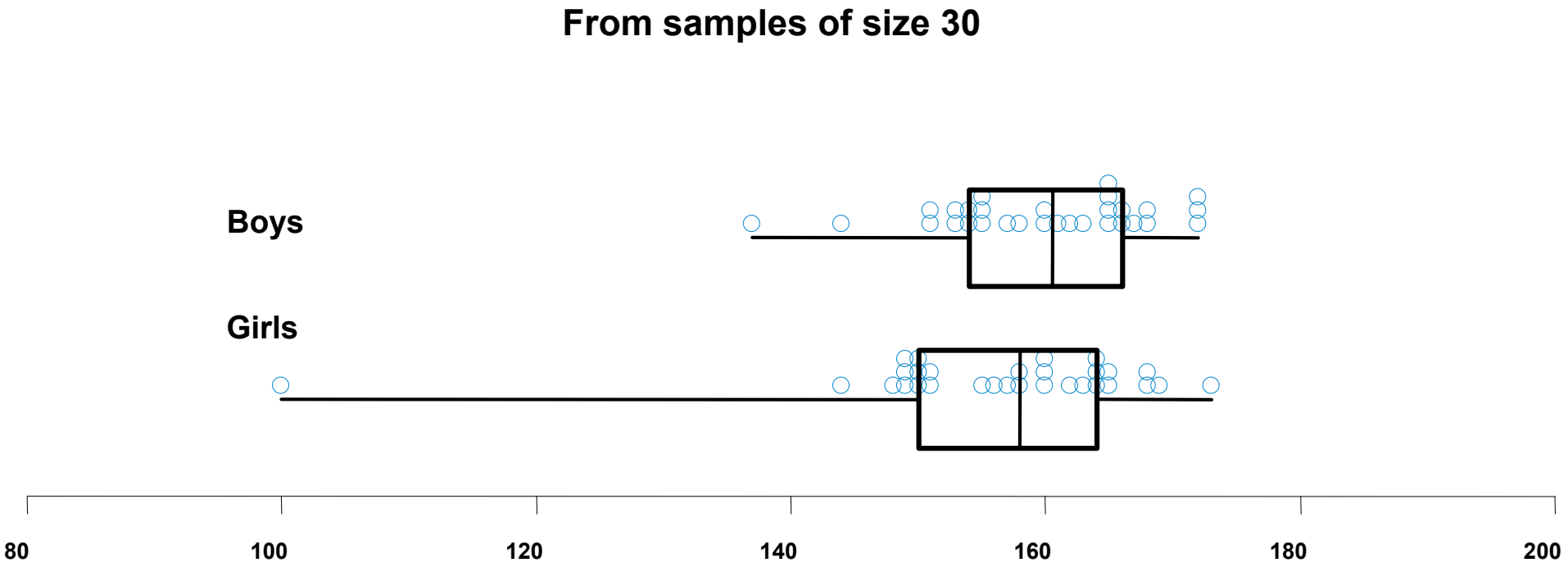
# Comparing heights of boys and girls at age 12



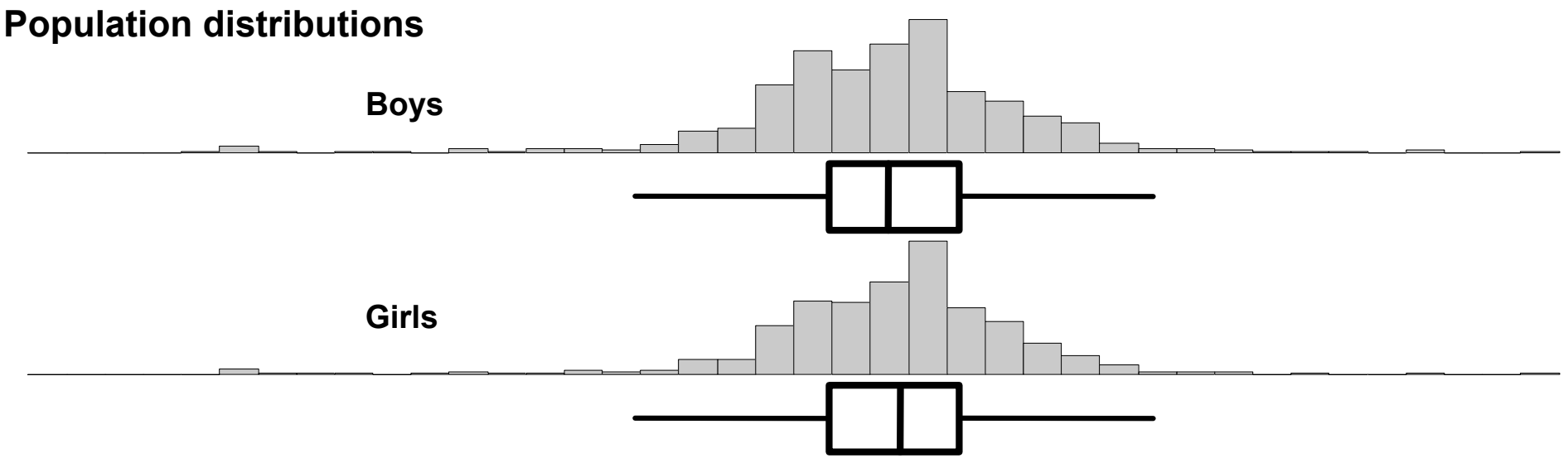
# Comparing heights of boys and girls at age 12



# Comparing heights of boys and girls at age 12



# Comparing heights of boys and girls at age 12

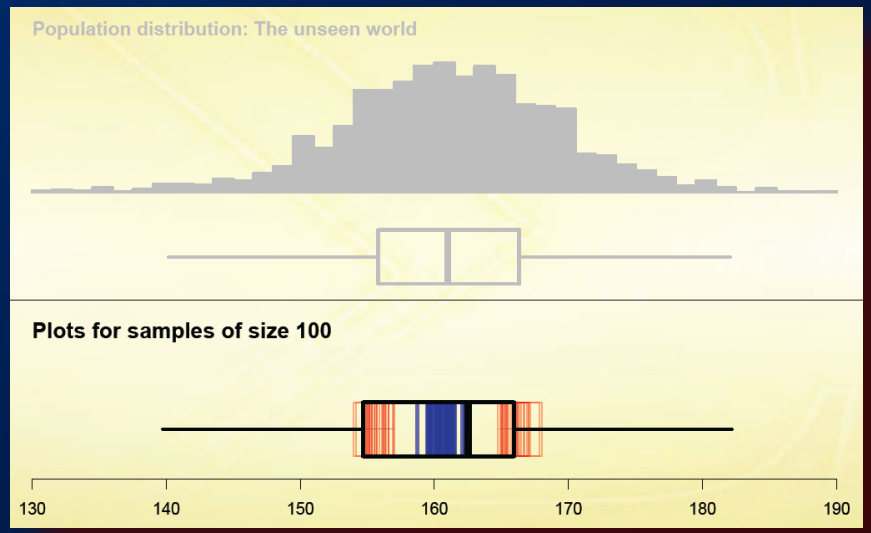


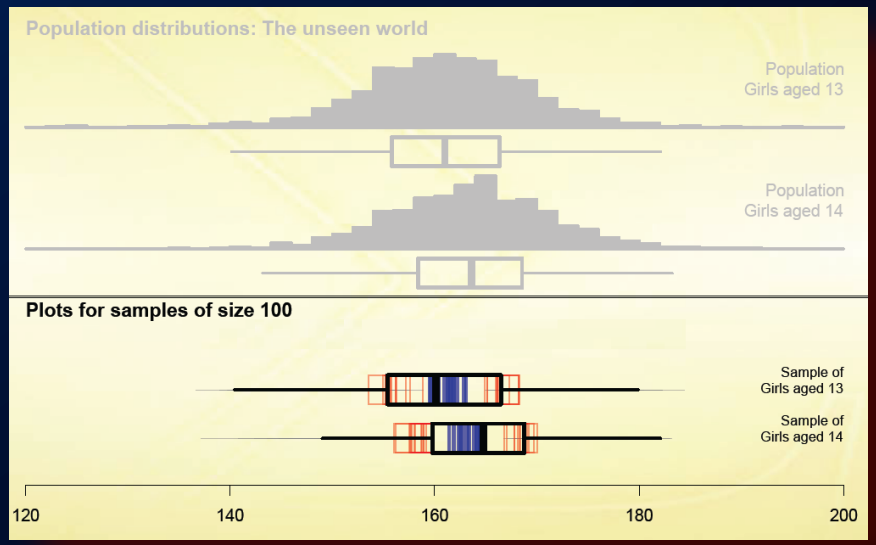
THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# Boxplots with a Memory

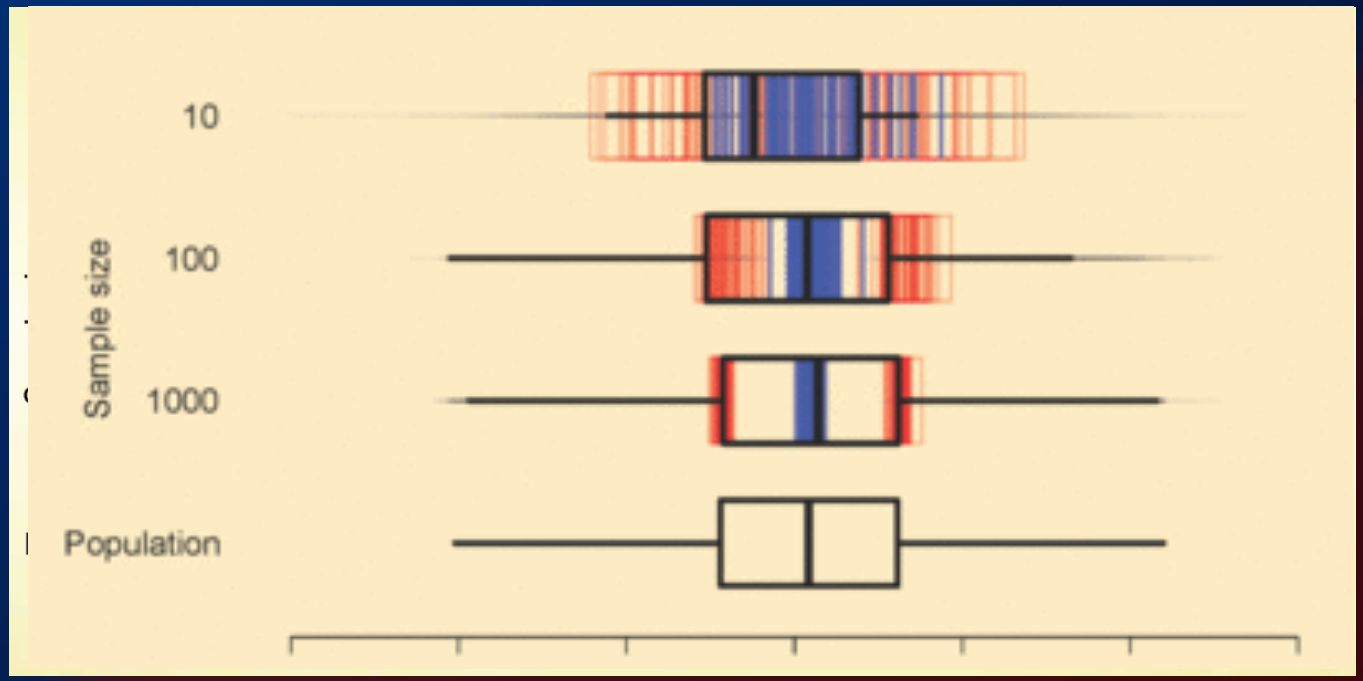
Play

- 1-sample build-up, n=30
- 2-sample build-up, n=30



# Effect of Sample Size



# Our Research – Method

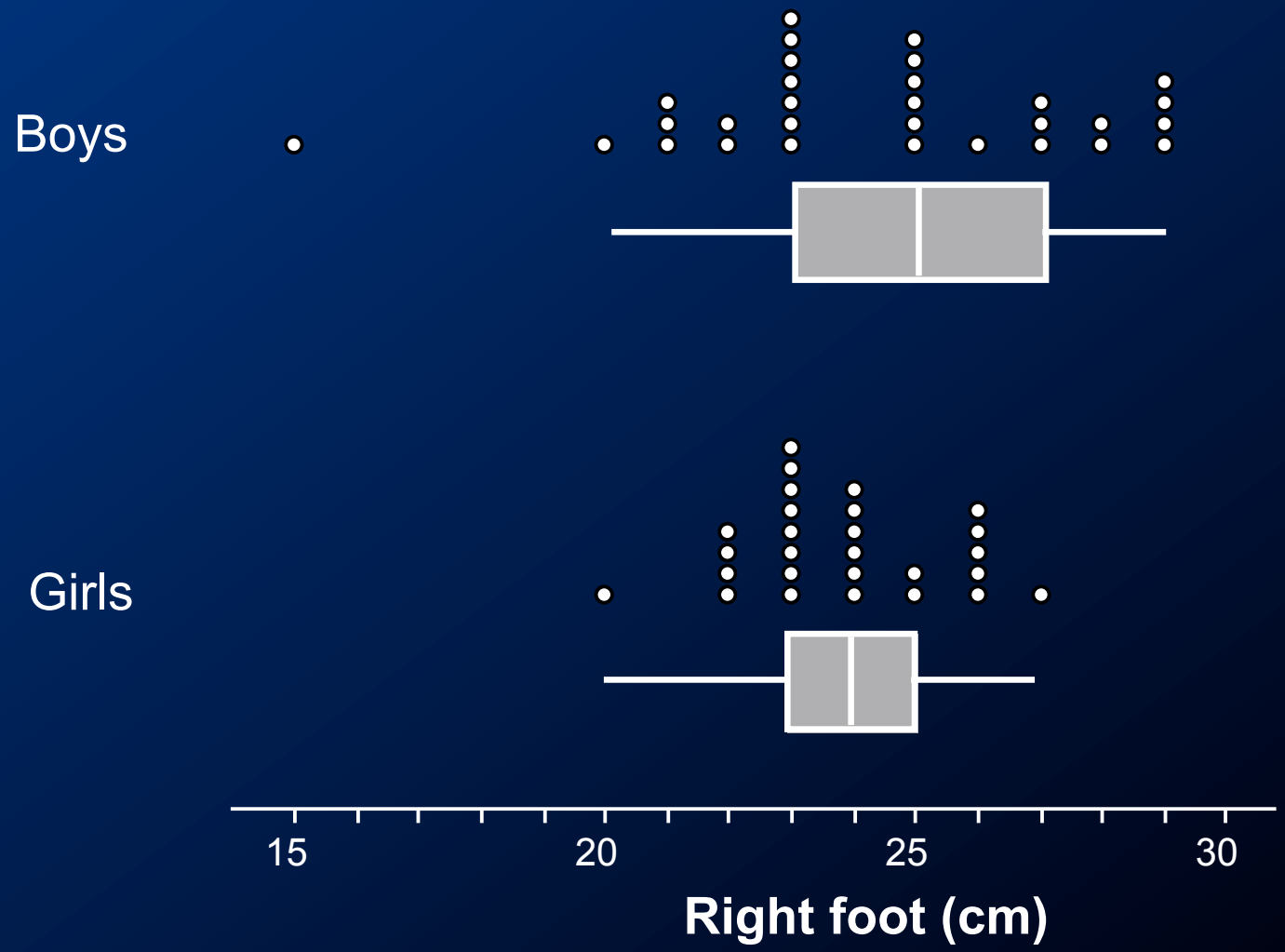
- Research Team
- Mixed methods
  - Design research (two cycles)
    - Materials designed to *engineer and support a new type of learning and reasoning*
  - Prepost tests (n≈100) from 2nd cycle
  - Prepost interviews (n=14) from 1st and 2nd cycles
- Participants (2nd cycle)
  - Four classes: Year 9 (two), Year 10, Year 11 from Decile 1, 4, 5, and 8 schools

    (Decile 1 = lowest socio-economic level, 10 = highest)

# Sample: What are students initially saying?

like those shopping stores that give you out free samples but you only get a little bit and it's kind of a sample

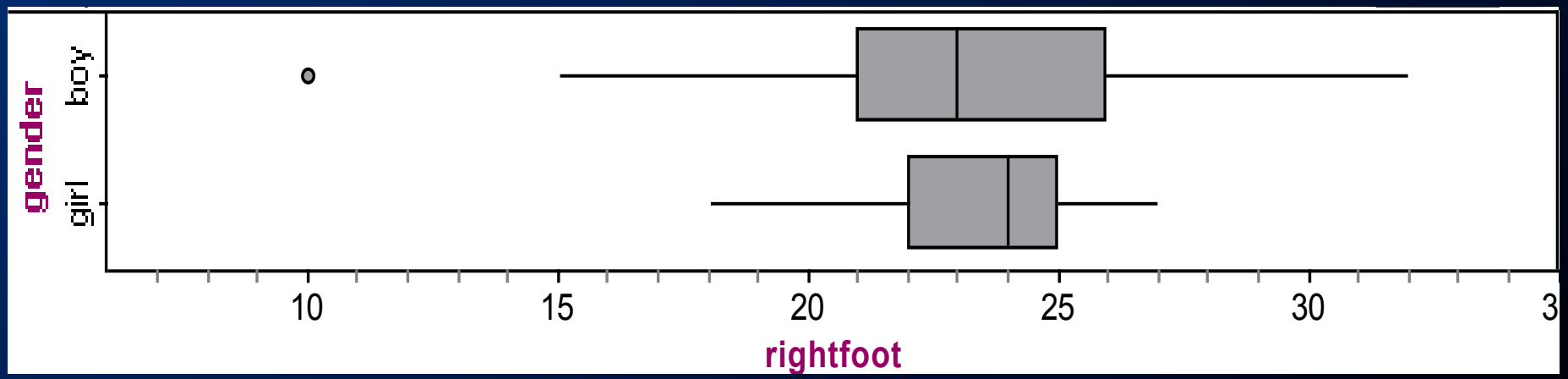
# Population: What are students initially saying?



# Population: What are students initially saying?

# What are students initially saying?

- Relationship of samples and populations
- Sampling variability
- Making a claim (n=30 for each gender)



THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# From hands-on to “movies”

Brief description of

# SEQUENCE OF INSTRUCTION

# Population ideas


Karekare College

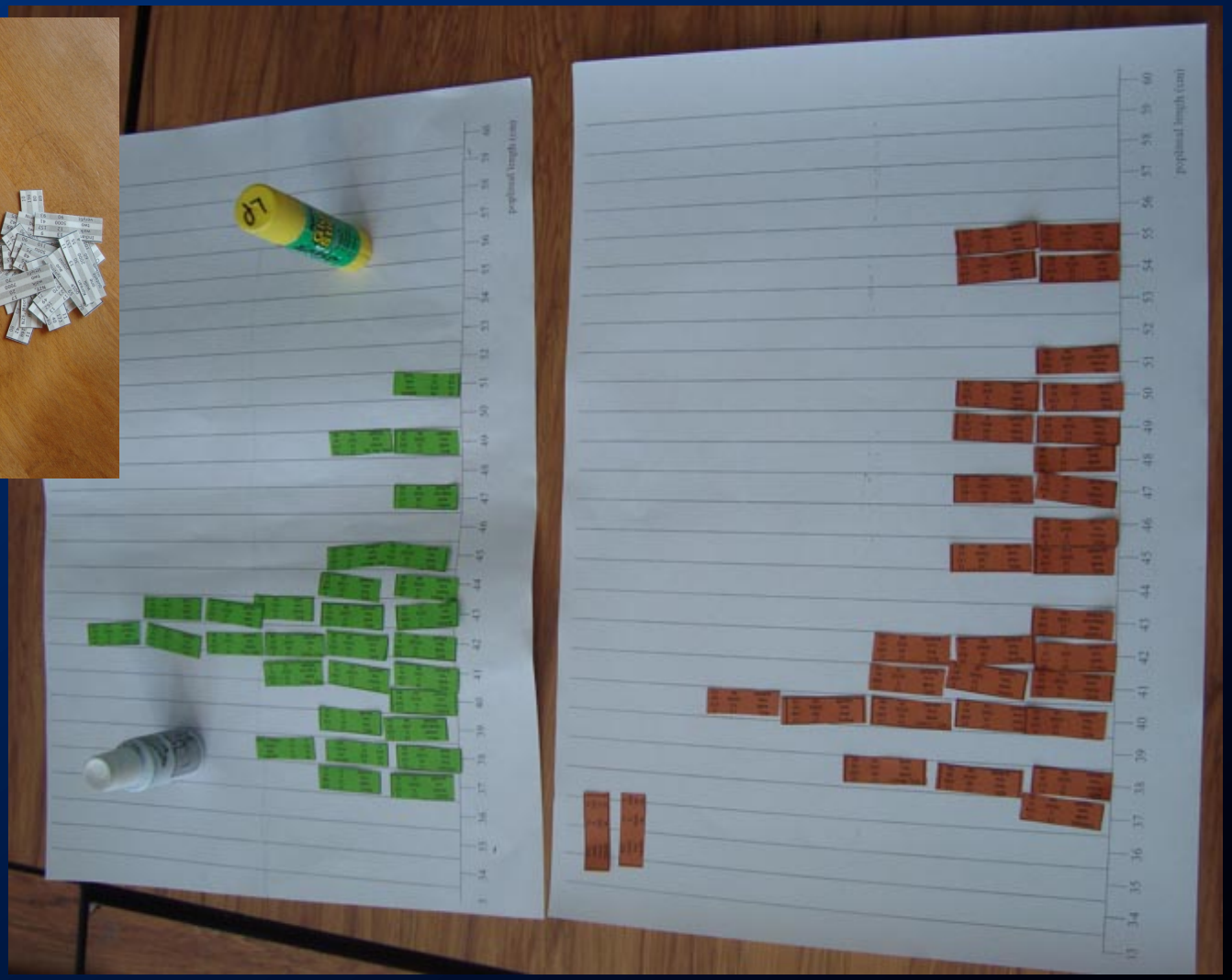
- Students selected from 2009 C@S
- 616 students
- 13 variables from C@S survey
- Each card represents a student
- Card colour indicates gender

# Karekare College DataCard

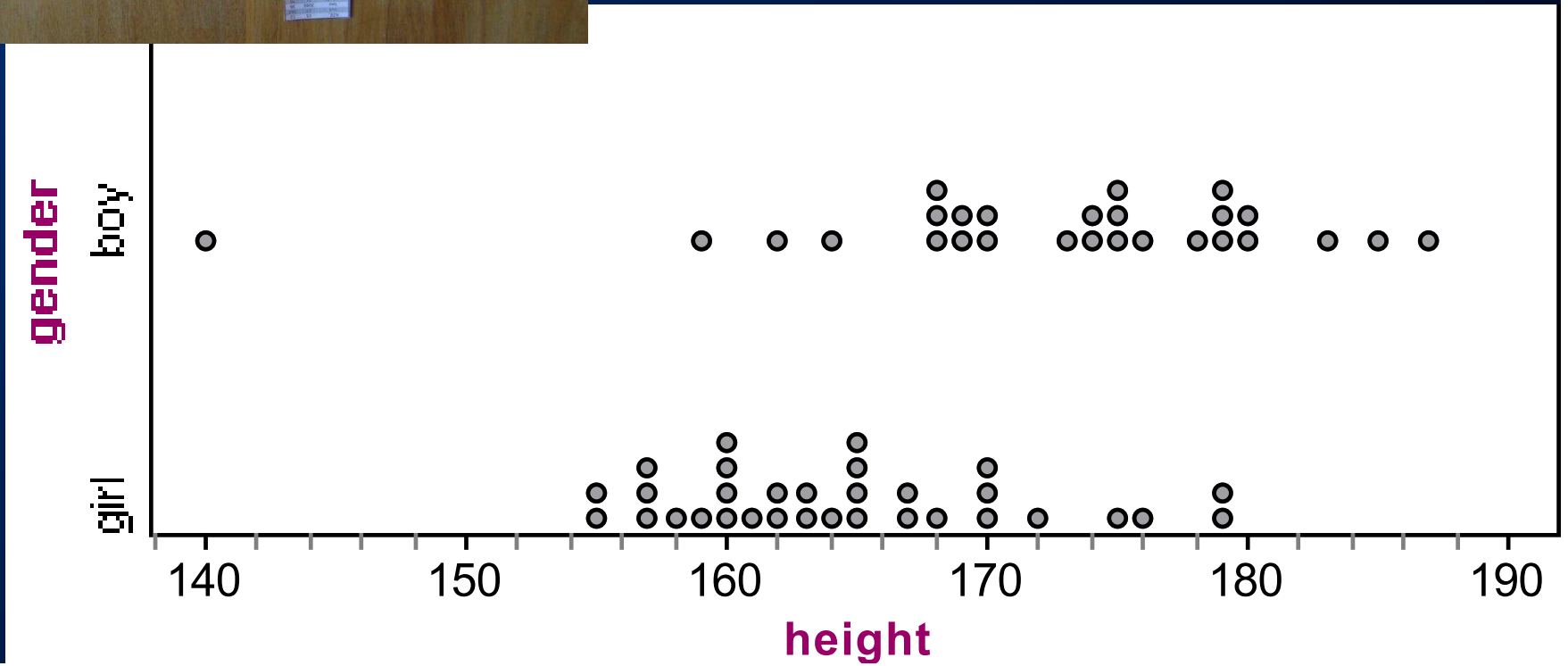
| | | |
|---|---|---|
| Ethnicity | Age | Year level |
| Transport to school | Time to school | Height |
| Way of carrying school bag | School bag weight | Popliteal length |
| Fitness level | Index finger length | Ring finger length |

THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

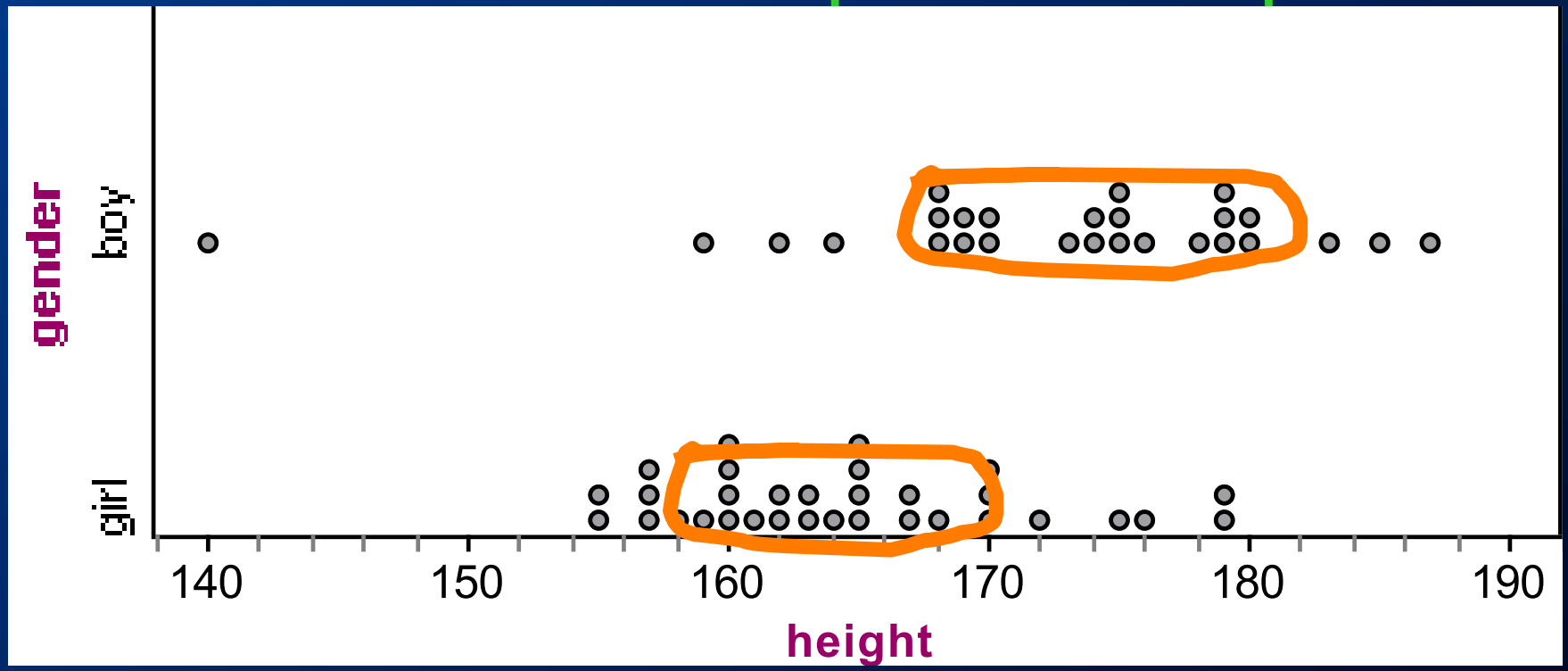
# Datacard plots & thinking about sampling variability

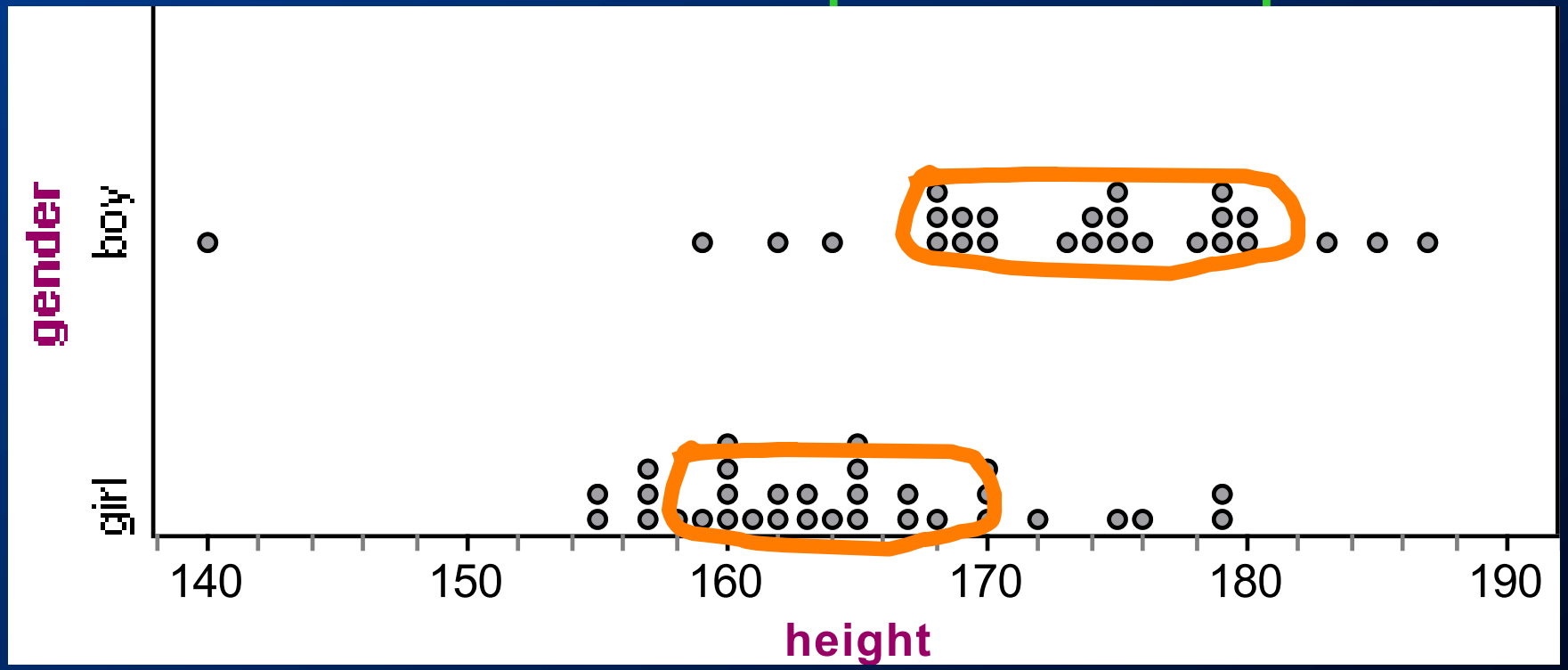
# Pre-work: From datacard plot to dot plot

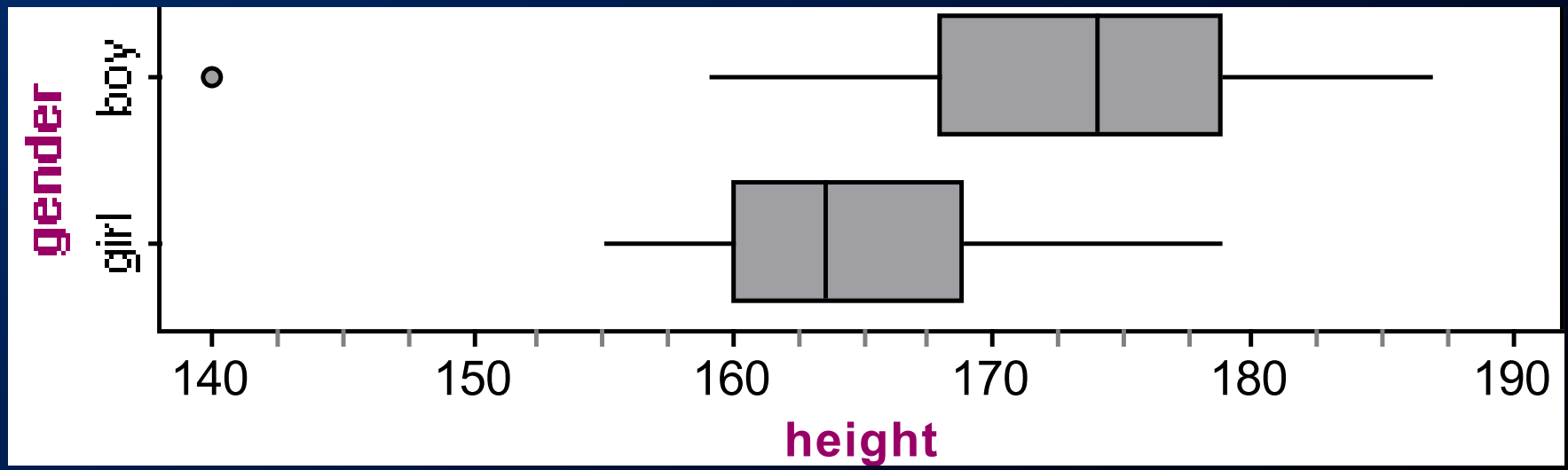


# Pre-work: From dot plot to box plot



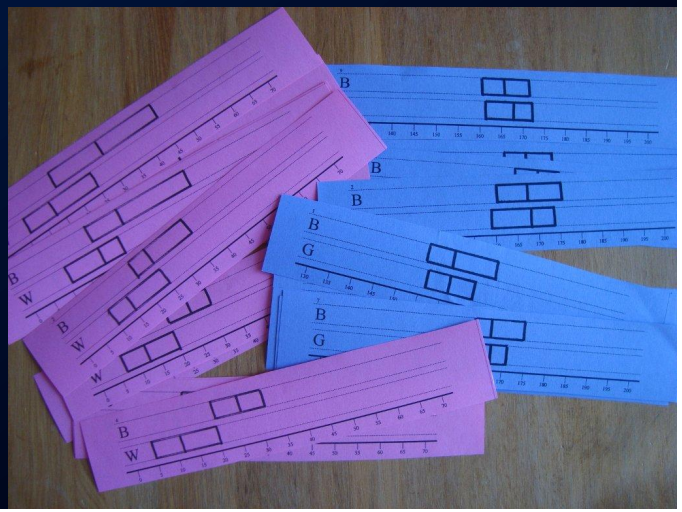
# Pre-work: From dot plot to box plot



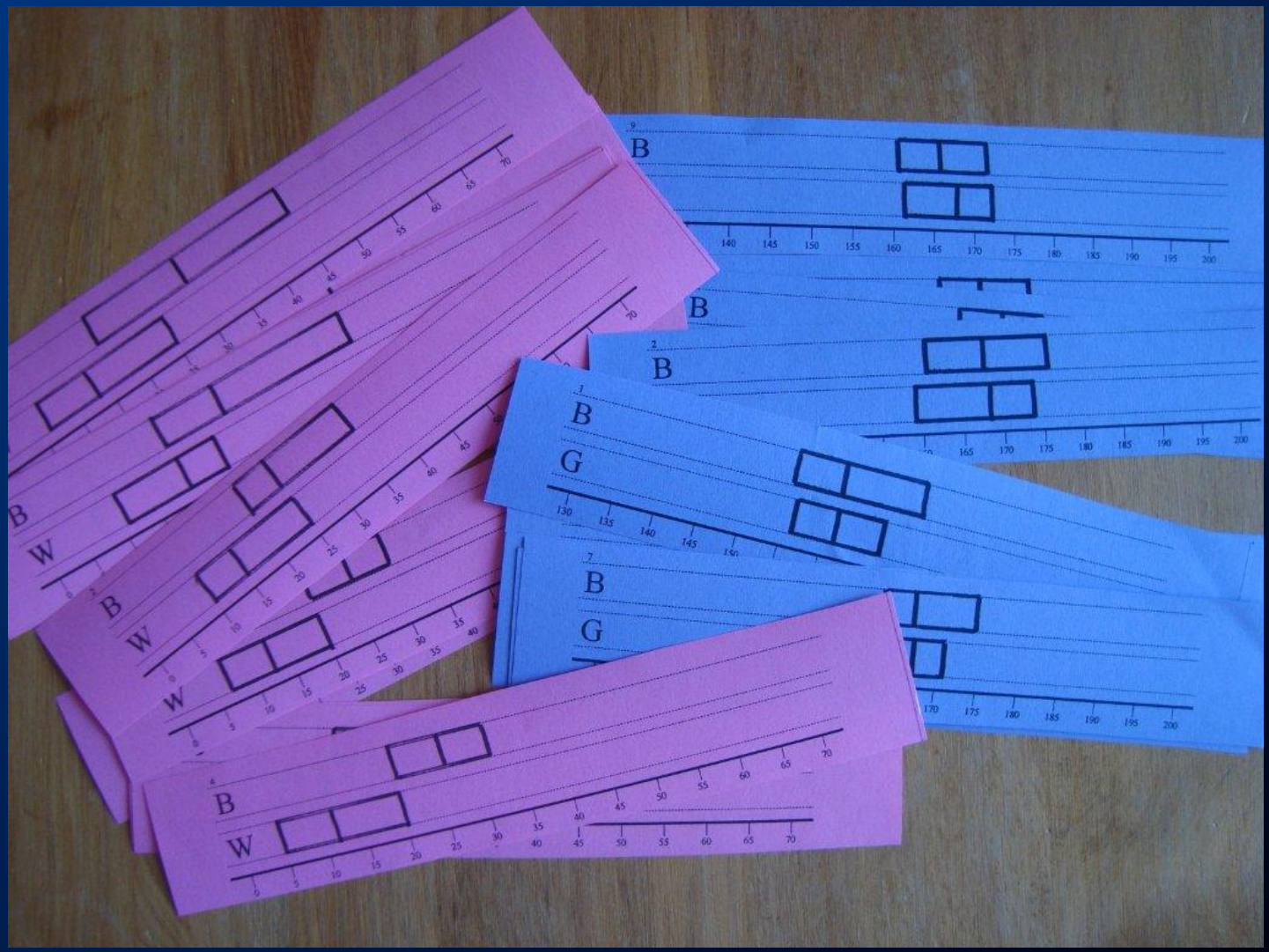



# Towards sampling variability: Questions for two situations

- Situation One:
  - Do Karekare College students who walk to school tend to get there faster than Karekare College students who take the bus?
- Situation Two
  - Do the heights of boys at Karekare College tend to be greater than the heights of girls at Karekare College?

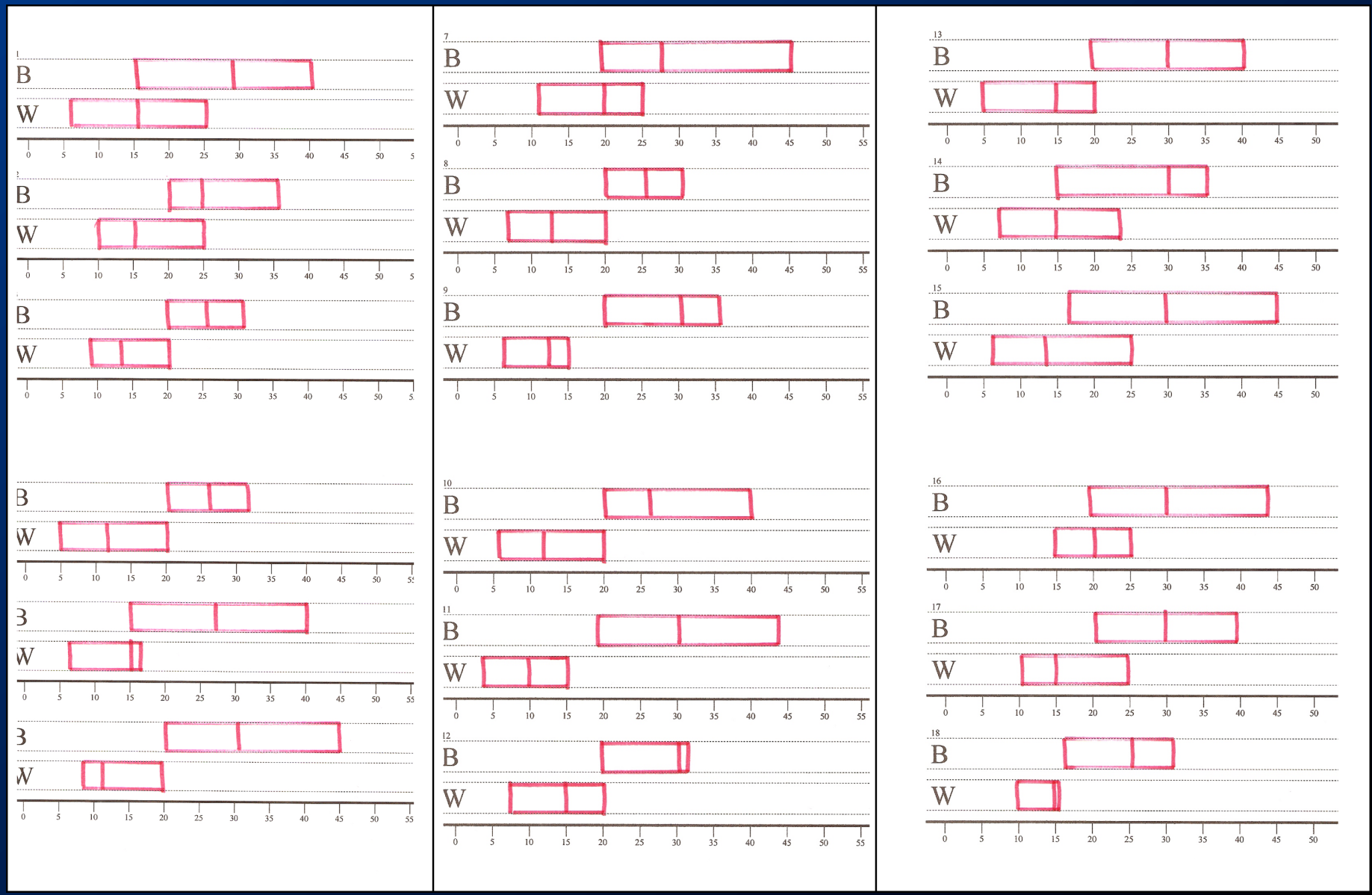


# Thinking about sampling variability & how to make a call or claim



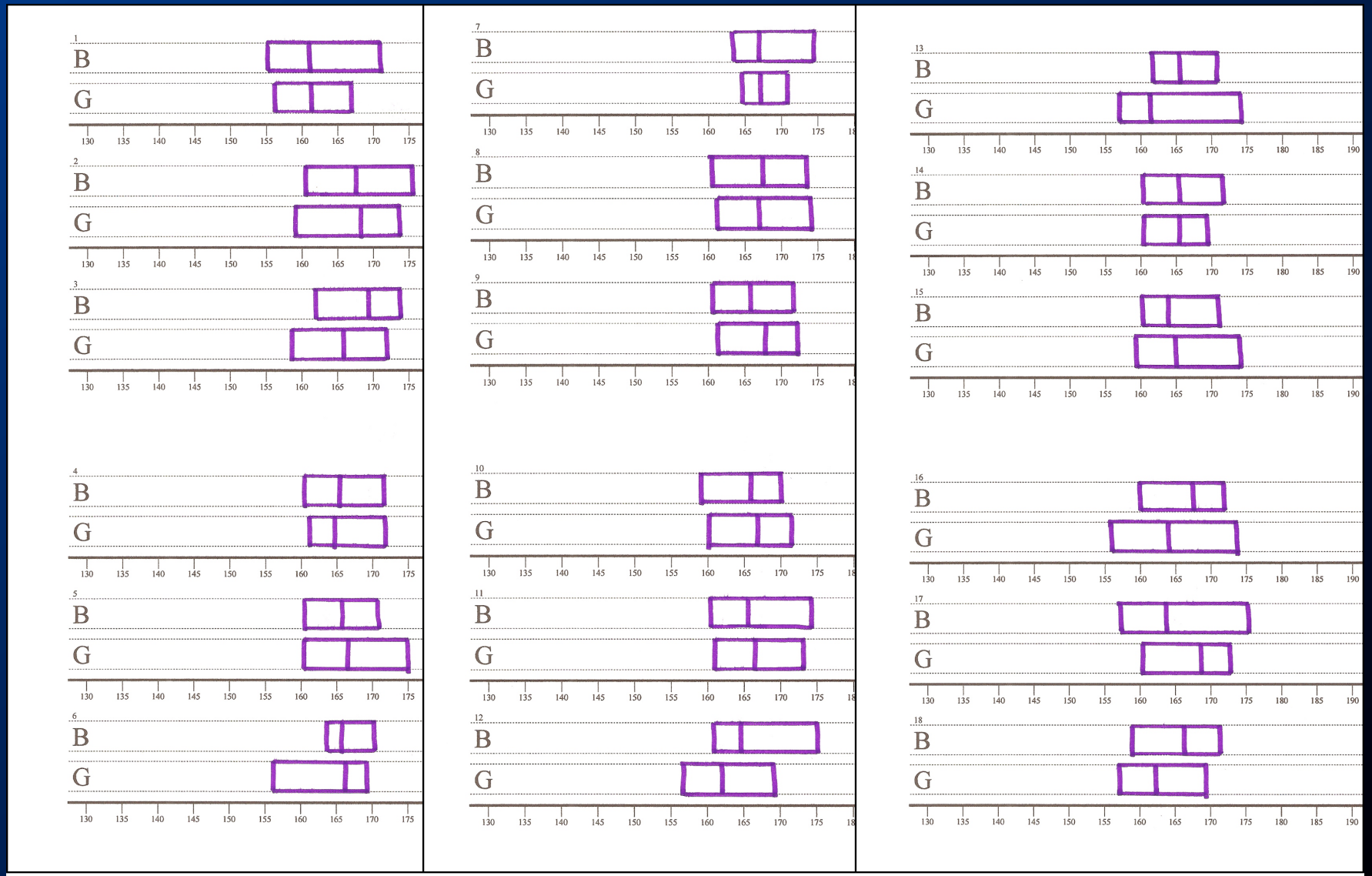
THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# Samples of bus & walking times to school



THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# Samples of boys and girls heights



THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

# Formulating guidelines for making a call

- **Small shift** and **large overlap** – not prepared to make a call, ('too close to call')
- **Large shift** and **small** or no **overlap** – make the call

How *large* does the shift have to be to make the call?



# Formulating guidelines for making a call

Make the call when

*At least one group's median is outside the box of the other group.*

otherwise,

it's too close to call.

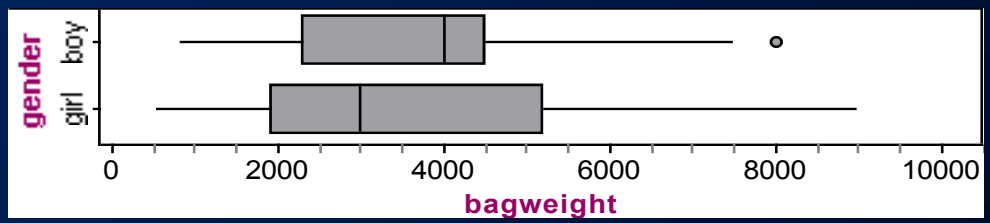
# Visualizing the suggested message

- Show animations
- Raise hands to indicate which median is higher.
- E.g.

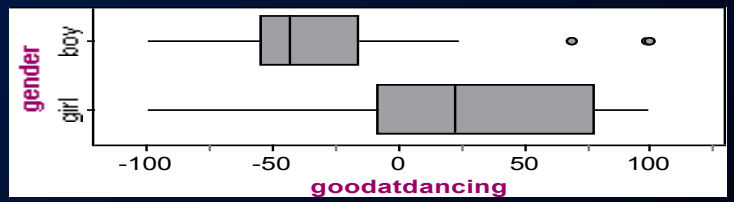
  boys higher – boys **up** the scale: **right** hand; girls higher – boys **down** the scale: **left** hand

# What are students saying after teaching?

- Sample
  - "It will hopefully tell you what is generally going to happen in the actual population."
- Sampling variability



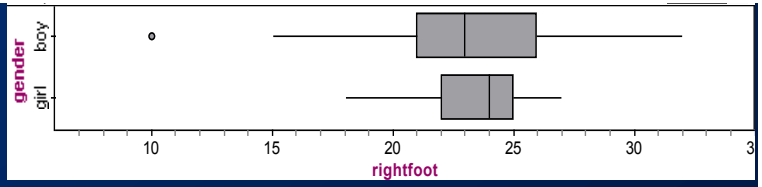

  - "I think that the medians may swap if another sample was taken"




"If I was able to take another sample the medians may move a little but would not swap (the girls would stay higher)"

THE UNIVERSITY OF AUCKLAND
DEPARTMENT OF STATISTICS

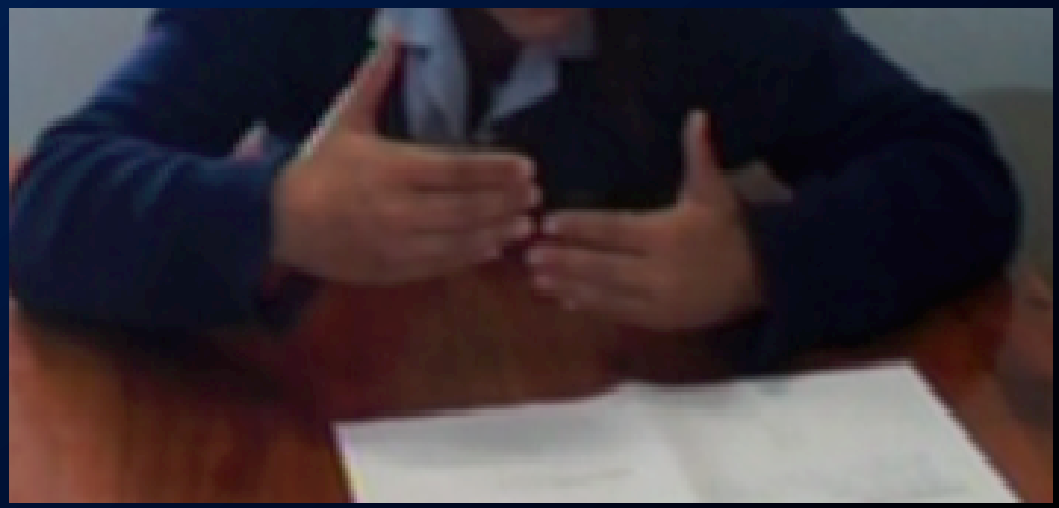
# Results: Prepost tests



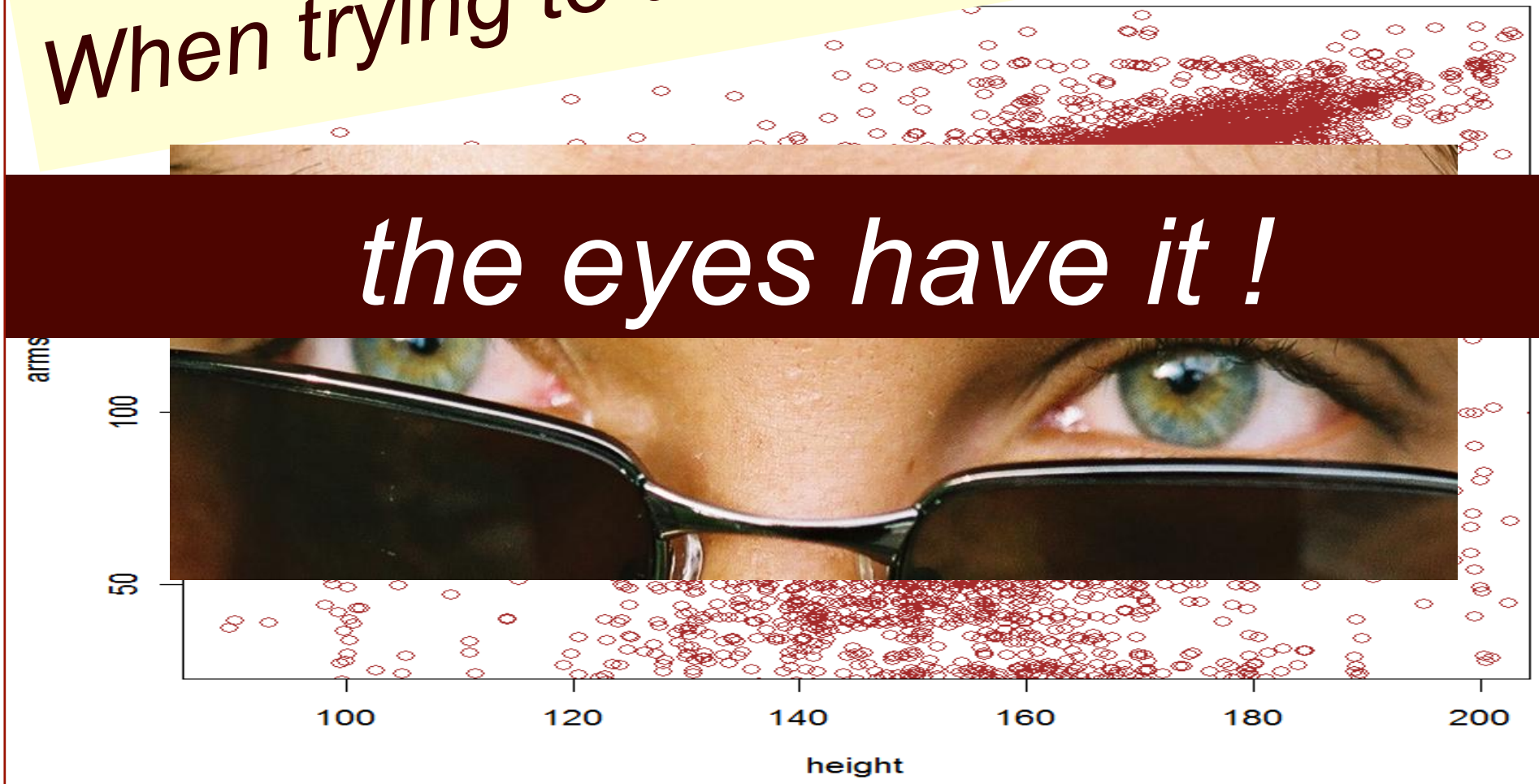

| Category | Score | Descriptor |
|---|---|---|
| Idiosyncratic (I): | 0 | No response or makes a statement not based on the data or any feature of the data. |
| Irrelevant evidence (IE): | 2 | Makes a call on any feature that appears bigger (e.g., maximum, box length, UQ). |
| Transitional (T): | 4 | Compares centres, the medians or central 50 percent. |
| Towards relevant evidence(TRE): | 6 | Makes correct call and fully verbalises one element of evidence and partially verbalises some other elements (shift, overlap, decision guideline and sampling variability) for justifying decision. |
| Relevant evidence (RE): | 8 | Makes correct call and fully verbalises at least two elements of evidence. |
| Full evidence (FE): | 10 | Fluent response with four elements of evidence and mentions samples, populations, and level of confidence where appropriate. |

| | | Post-test Average | | | | | |
|---|---|---|---|---|---|---|---|
| | | **I** | **IE** | **T** | **TRE** | **RE** | **Total** |
| **Pre-test Average** | **I** | 0 | 4 | 9 | 2 | 0 | 15 |
| | **IE** | 0 | 5 | 22 | 16 | 5 | 48 |
| | **T** | 0 | 1 | 5 | 7 | 13 | 26 |
| | **TRE** | 0 | 0 | 0 | 0 | 2 | 2 |
| | **RE** | 0 | 0 | 0 | 0 | 0 | 0 |
| **Total** | | 0 | 10 | 36 | 25 | 20 | 91 |

# What images did *students* have?

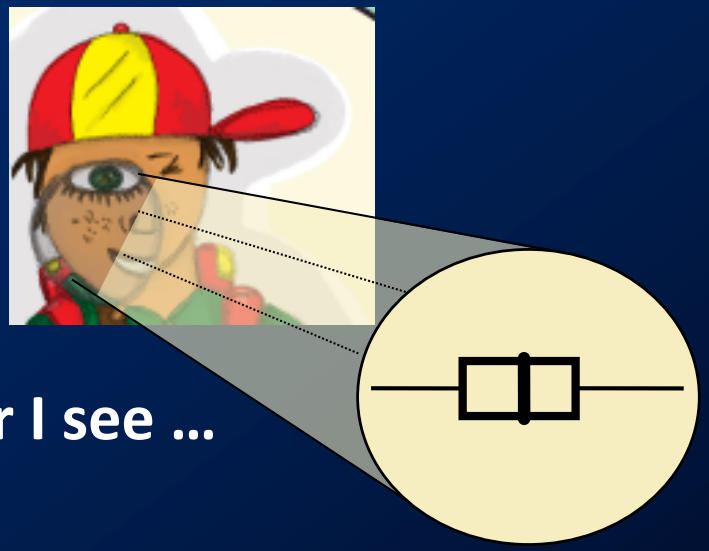


# When trying to understand data …



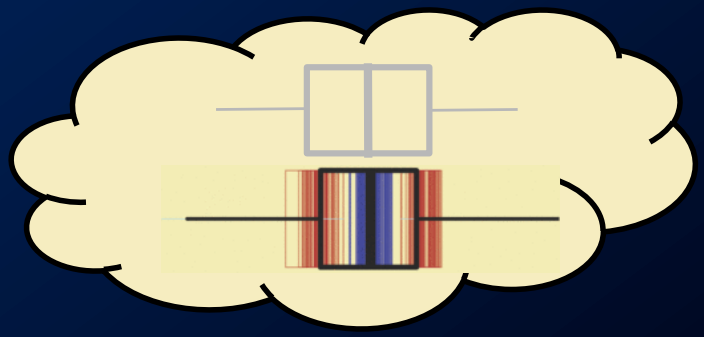

## *the eyes have it !*

# Want to plant a reflex of the form …

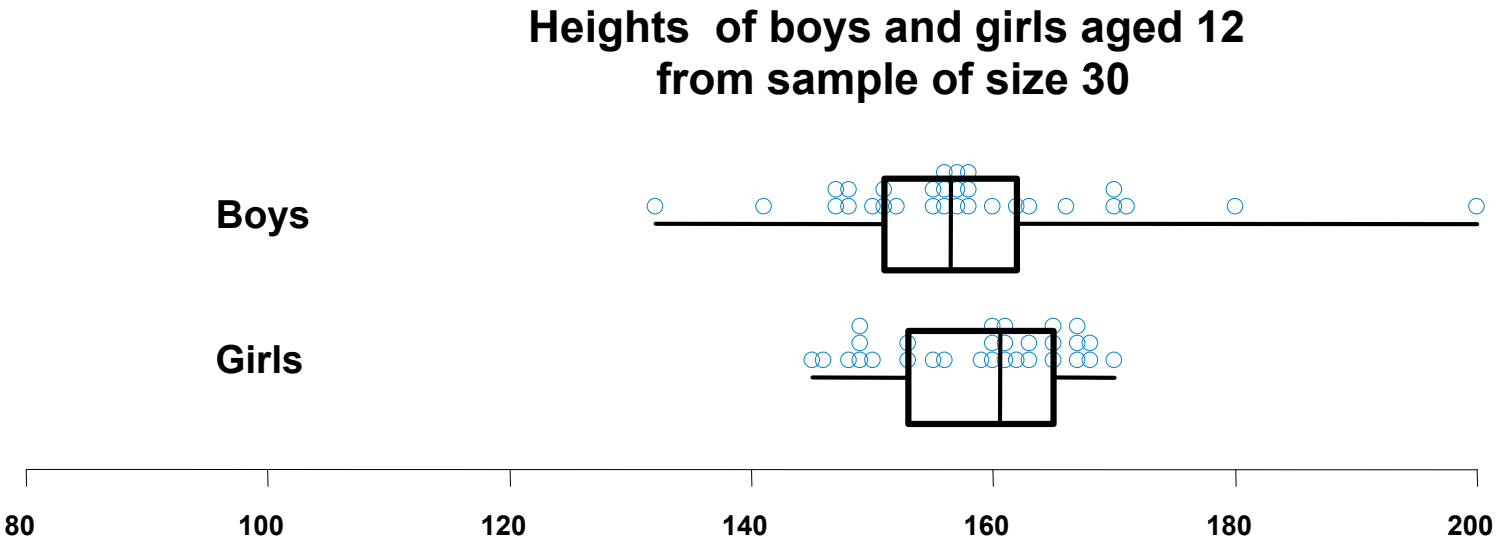


"Whenever I see …

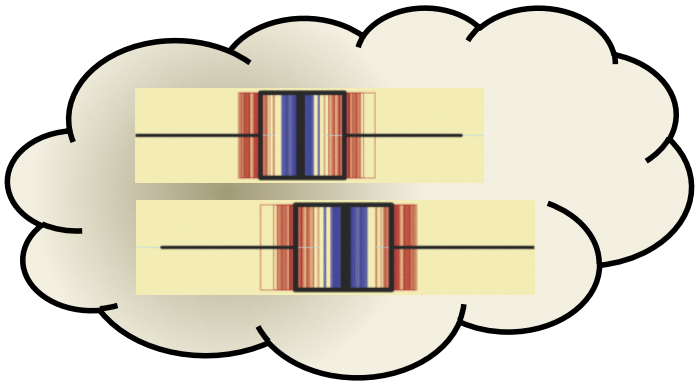
"I remember …



# Making a call




What should I say?



Dare I make the claim,

"Girls tend to be taller than boys

*back in the populations*"

# Our World has Changed in Statistics

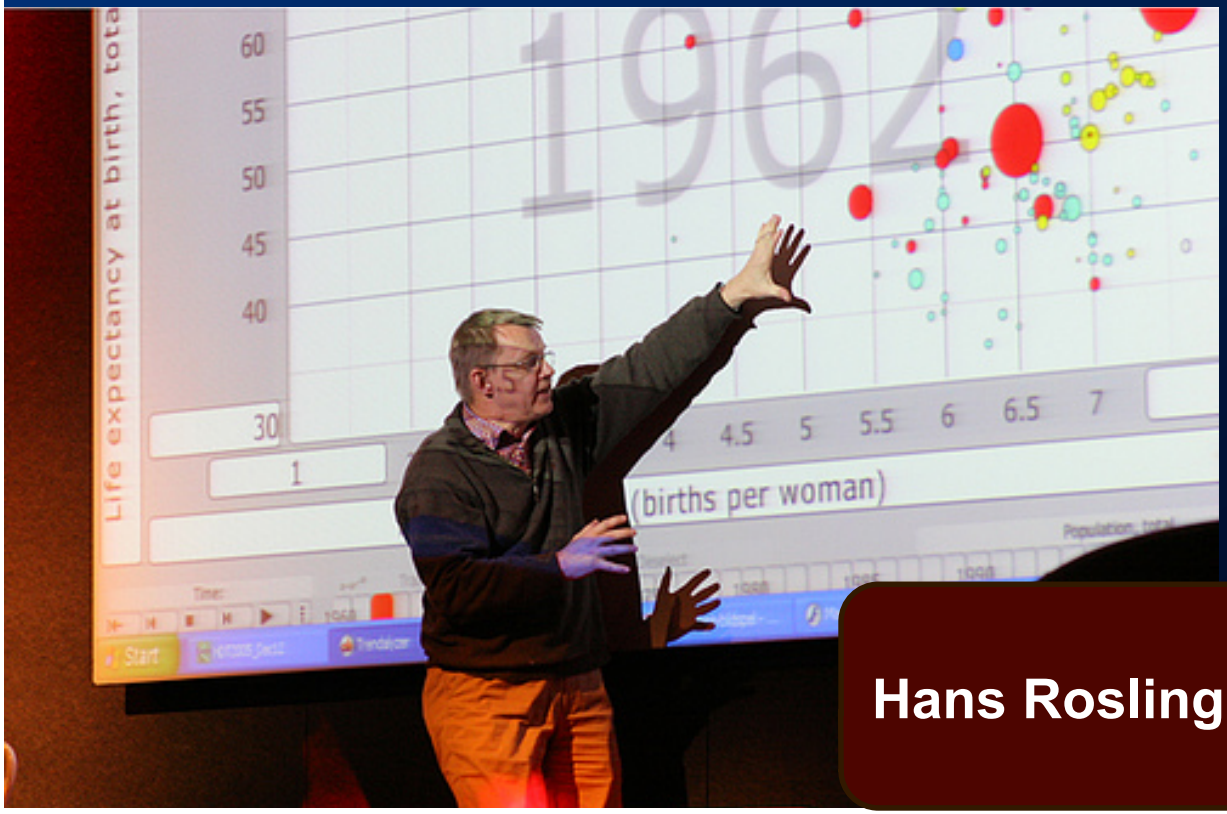
## Technology – a seismic shift

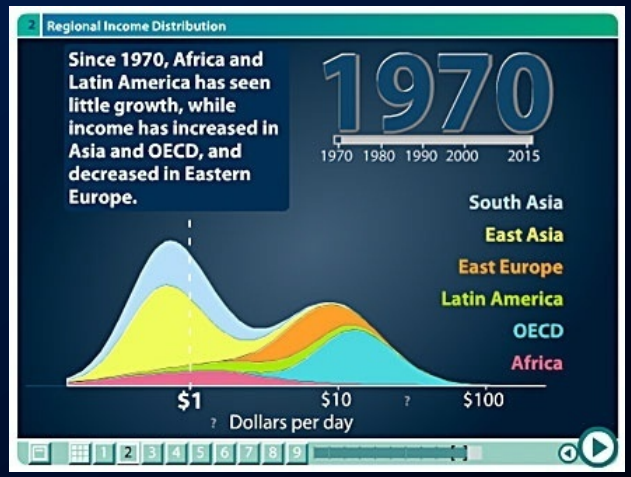


Christchurch Earthquakes 2010 and 2011

# Our World has Changed in Statistics

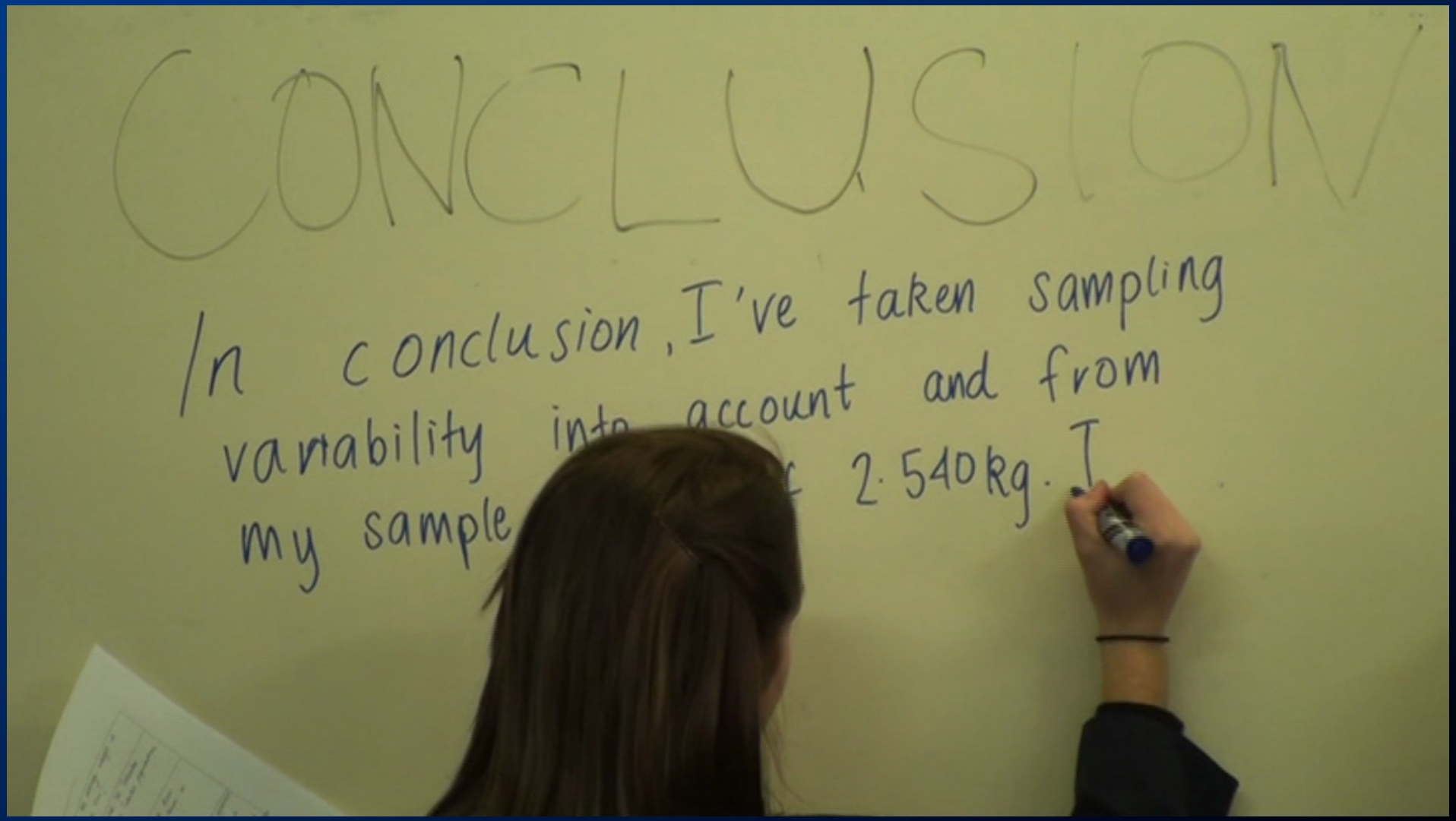
No longer an emphasis on constructing plots
Now an emphasis on ***reasoning from data***
Small part of that reasoning involves ***inference***
***Inferential concepts*** are now ***accessible*** to all students




**Hans Rosling**'s 2006 and 2007 TED talks

# THANK YOU!!




Resources available: www.censusatschool.org.nz/2009/informal-inference/