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Valószínűség-maximalizálás belső közelítéssel

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Outline

Probabilistic optimization problems

Classic and recent solution approaches

A variant of a classic approach: epi-approximation
- Approximating the epigraph of the probabilistic function

Computational study

Theoretical justification of heuristic adjustments

Working with gradient estimates

Problem formulation

$\boldsymbol{\xi}$ an n-dimensional random vector, with known distribution.

Let $F(.)$ denote the cumulative distribution function.

$\boldsymbol{x} \in \mathbb{R}^m$ decision variables

$T\boldsymbol{x}$ a linear function of the decision variables

Probabilistic function

$$\mathrm{P}(T\boldsymbol{x} \geq \boldsymbol{\xi}) = F(T\boldsymbol{x})$$

Probability maximization

$$\max\ F(T\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{x} \in X.$$

Joint probabilistic constraint

$$\min\ \boldsymbol{c}^T\boldsymbol{x} \quad \text{subject to} \quad \boldsymbol{x} \in X, \qquad F(T\boldsymbol{x}) \geq p.$$

Equivalent formulation for joint probabilistic constraint:

$$\min\ \boldsymbol{c}^T\boldsymbol{x} \quad \text{subject to} \quad \boldsymbol{x} \in X, \qquad T\boldsymbol{x} \in \mathcal{L},$$

where $\mathcal{L} = \{\, \boldsymbol{z} \mid F(\boldsymbol{z}) \geq p \,\}.$

Early application

strategic planning model for the Hungarian energy sector
Prékopa, Ganczer, Deák, Patyi (1980).

Characterization and convexity statements, Prékopa (1970-1973):

Assume $\boldsymbol{\xi}$ has a continuous distribution with a log-concave density function.

$\Rightarrow$ the cumulative distribution function $F(.)$ is log-concave

$\Rightarrow$ the probabilistic function $\boldsymbol{x} \mapsto F(T\boldsymbol{x})$ is log-concave.

Solution approaches

Feasible direction method in the convex level set $\mathcal{L}$.

Prékopa, Ganczer, Deák, Patyi (1980).

Cutting-plane methods approximating the convex level set $\mathcal{L}$.

Prékopa and Szántai (1978), Szántai (1988), Mayer (1998),

Henrion and associates (2000 -).

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Henrion and associates (2000 -).

- Efficiency due to reusing former gradient information.
- Difficulty: gradient computation is noisy.
  Practicable implementations require
  sophisticated tolerance handling.

Other solution approaches

Uncertain convex programs,

Campi, Calafiore, Garatti, Caré (2005 -)

Sample Average Approximation, integer programming formulations,

Ahmed, Luedtke, Nemhauser (2007 -)

Cone programming,

Cheng, Gicquel and Lisser (2012).

Classic dual approach

Idea: to build an inner approximation of the level set $\mathcal{L}$.

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Definition (Prékopa 1990): z is a p-efficient point iff

$$\begin{cases} z \in \mathcal{L} \quad \text{and} \\ \text{there exists no } z' \in \mathcal{L} \text{ such that } z' \leq z,\, z' \neq z. \end{cases}$$

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Given p-efficient points $\boldsymbol{z}_1, \ldots, \boldsymbol{z}_k$,

$$\operatorname{conv}(\boldsymbol{z}_1, \ldots, \boldsymbol{z}_k) + \mathbb{R}^n_+ \qquad \text{is an inner approximation of } \mathcal{L}.$$

Solution metods

Prékopa, Vizvári, Badics (1998):
set of p-efficient points generated before optimization,

Dentcheva, Prékopa, Ruszczyński (2000):
primal-dual method (cone-generation),

Dentcheva, Lai, Ruszczyński (2004), Dentcheva and Martinez (2013).

Cone generation Dentcheva, Prékopa, Ruszczyński (2000)

Primal problem formulated by splitting variables:

$$(\mathcal{P}) \qquad \min\ \boldsymbol{c}^T\boldsymbol{x} \quad \text{subject to} \quad T\boldsymbol{x} = \boldsymbol{z}, \quad \boldsymbol{x} \in X, \quad \boldsymbol{z} \in \mathcal{L}.$$

Lagrangian dual problem obtained by relaxing the constraint $T\boldsymbol{x} = \boldsymbol{z}$:

$$(\mathcal{D}) \qquad \max_{\boldsymbol{u}}\ D(\boldsymbol{u}) \quad \text{where} \quad D(\boldsymbol{u}) = \min_{\boldsymbol{x} \in X} (\boldsymbol{c}^T - \boldsymbol{u}^T T)\boldsymbol{x} \ + \ \min_{\boldsymbol{z} \in \mathcal{L}} \boldsymbol{u}^T \boldsymbol{z}.$$

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Solution method:

– from dual viewpoint: cutting-plane method for $D()$,

– from primal viewpoint: column generation method.

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New cuts/columns are improving p-efficient points.

These are found by solving subproblems $\min_{\boldsymbol{z} \in \mathcal{L}} \boldsymbol{u}^T \boldsymbol{z}$.

An alternative dual approach: epi-approximation

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Convex formulation: $-\ln F(z) \leq -\ln p$

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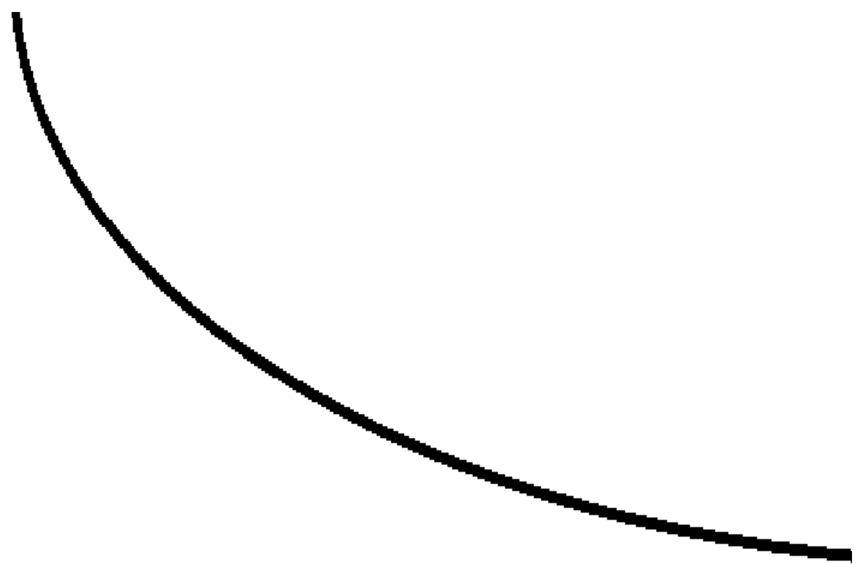

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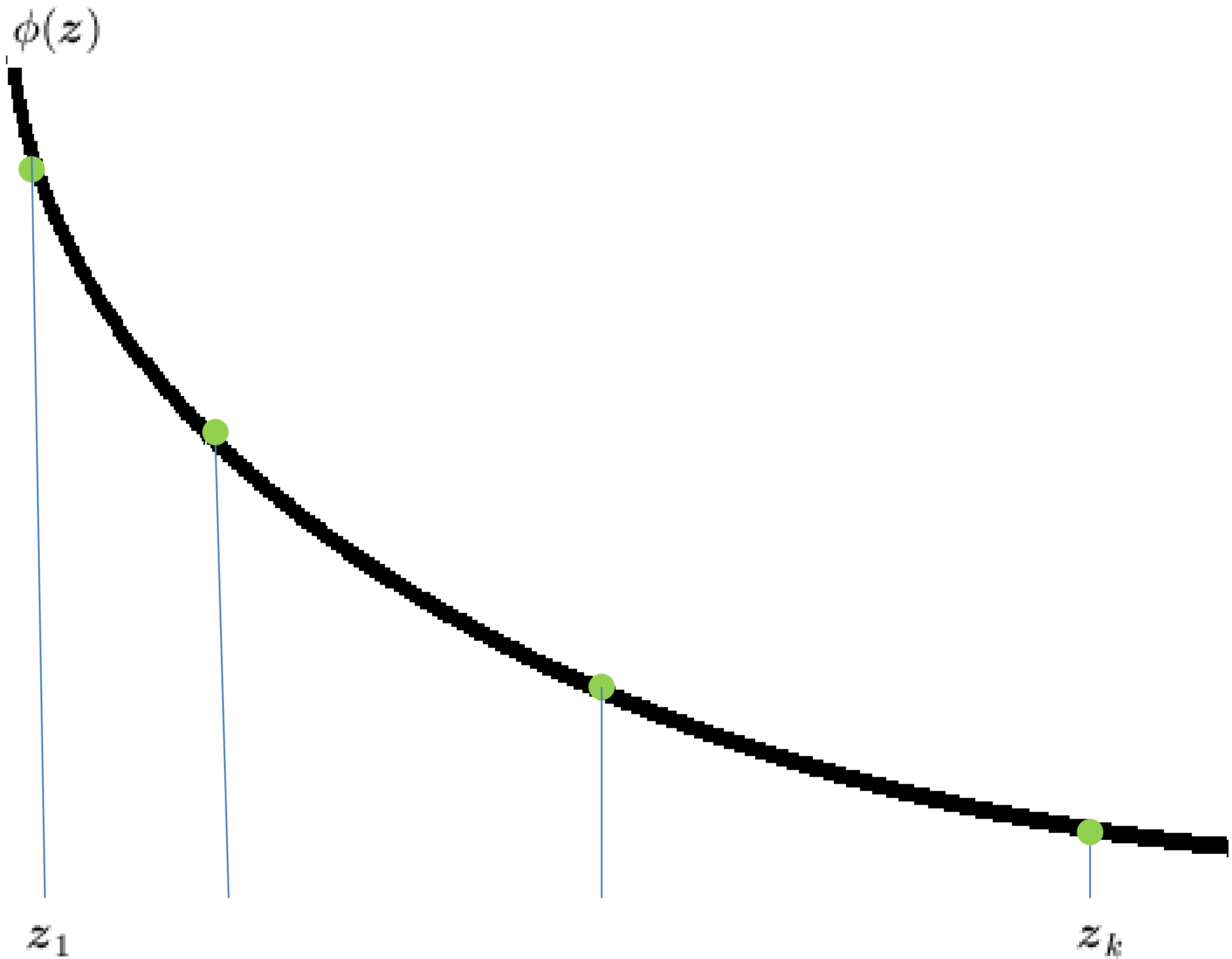

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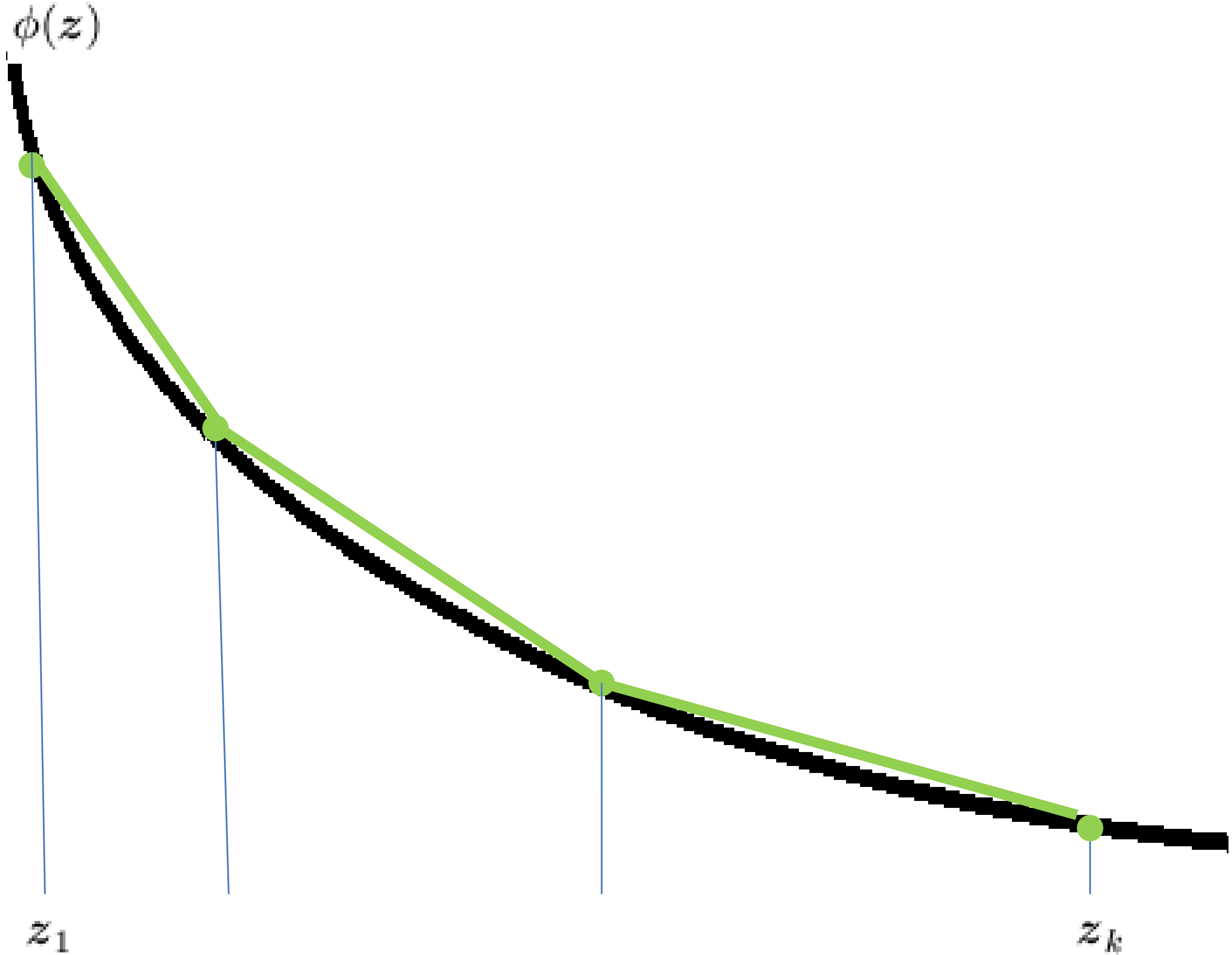

Epi-approximation to probability maximization

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Epi-approximation computational study

Problems

Normal distribution, dimension up to 15.

Gradients of the distribution function can be computed componentwise:

$$\frac{\partial F(z_1,\ldots,z_n)}{\partial z_i} = F(z_1,\ldots,z_{i-1},z_{i+1},\ldots,z_n \mid z_i)\, f_i(z_i) \qquad (i = 1,\ldots,n).$$

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Implementation: column generation scheme

Master problem solved by CPLEX simplex, version 12.6.3.

Subproblems solved by a steepest descent method.

$$\min_{\boldsymbol{z}} \left\{ \phi(\boldsymbol{z}) - \boldsymbol{u}^T \boldsymbol{z} \right\} \quad \text{where} \quad \phi(\boldsymbol{z}) = -\log F(\boldsymbol{z}).$$

$F(\boldsymbol{z})$ function values and gradients computed by Genz's code.
A gradient component can be computed from
an appropriate (n-1)-dimensional normal distribution function value.

Experience

Code proved reliable and robust.

Number of iterations depends on problem dimension, and on the optimal probability achieveable.

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Heuristic adjustment

Aim: balancing different efforts:

– solving and resolving master problem (CPLEX),
– solving subproblems (Genz code).

Approximate solution of subproblems.

A single line search made in each steepest descent procedure,
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A single line search made in each steepest descent procedure, and even this line search is approximate.

Outcome

Effort spent in a subproblem substantially decreased

Number of master iterations did not increase significantly.

Theoretical justification of heuristic adjustment

An ideal convex programming problem

$\mathcal{F} \; = \; \min_{\boldsymbol{z}} f(\boldsymbol{z})$

Assume the function $f(\boldsymbol{z})$ is twice continuously differentiable,

and there are $\alpha, \omega \in \mathbb{R}$ $(0 < \alpha \leq \omega)$ such that

$$\alpha I \; \preceq \; \nabla^2 f(\boldsymbol{z}) \; \preceq \; \omega I \qquad (\boldsymbol{z} \in \mathbb{R}^n).$$

Here the relation $U \preceq V$ means that $V - U$ is positive semidefinite.

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A well-known convergence theorem

We minimize $f(\boldsymbol{z})$ using a steepest descent method.

Starting from $\boldsymbol{z}^0$, let $\boldsymbol{z}^1, \ldots, \boldsymbol{z}^j, \ldots$ denote the iterates.

Then we have

$$f\left(\boldsymbol{z}^j\right) - \mathcal{F} \leq \left(1 - \tfrac{\alpha}{\omega}\right)^j \left[\, f\left(\boldsymbol{z}^0\right) - \mathcal{F}\,\right].$$

convergence theorem

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Application to our column generation scheme

New columns are found by solving subproblems

$$\min_{\boldsymbol{z}} \;\left\{\, \phi(\boldsymbol{z}) - \boldsymbol{u}^T \boldsymbol{z} \;\; + \text{constant} \,\right\}.$$

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$$\mathcal{F} = \min_{\boldsymbol{z}} \left\{ \underbrace{\phi(\boldsymbol{z}) - \boldsymbol{u}^T \boldsymbol{z} \;\; + \text{constant}}_{f(\boldsymbol{z})} \right\}.$$

Corollary of the theorem

We can find a starting point $\boldsymbol{z}^0$ such that

$$\boldsymbol{u}^T \boldsymbol{z}^j - \phi(\boldsymbol{z}^j) \;\; - \text{constant} \;\ge\; \left[1 - \left(1 - \tfrac{\alpha}{\omega}\right)^j\right] (-\mathcal{F}).$$

holds with iterate $\boldsymbol{z}^j$ obtained by j line search iterations.

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We can find a starting point $\boldsymbol{z}^0$ such that

$$\underbrace{\boldsymbol{u}^T \boldsymbol{z}^j - \phi(\boldsymbol{z}^j) \;\; - \text{constant}}_{\text{reduced cost of column } \boldsymbol{z}^j} \geq \left[1 - \left(1 - \tfrac{\alpha}{\omega}\right)^j\right] \underbrace{(-\mathcal{F})}_{\text{largest possible reduced cost value}}.$$

Consider the master problem as a LP problem.

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Consider the master problem as a LP problem.

Even for a moderately large j, the iterate $\boldsymbol{z}^j$ is
a fairly good improving column in the column generation scheme.

On α, ω

Requirement: $\alpha I \preceq \nabla^2 \phi(\boldsymbol{z}) \preceq \omega I$

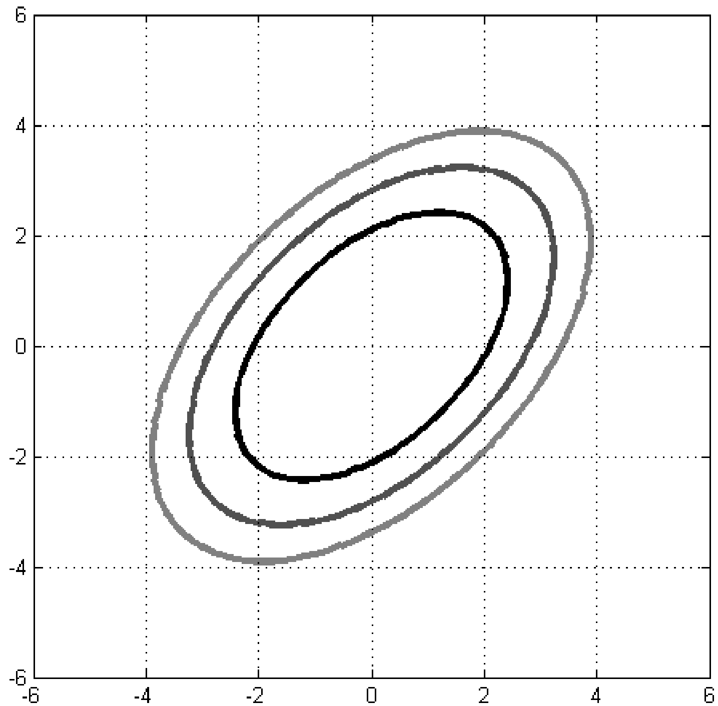
In the present model, we have $\phi(\boldsymbol{z}) = -\ln F(\boldsymbol{z})$.

Requirement does not hold for every $\boldsymbol{z}$.

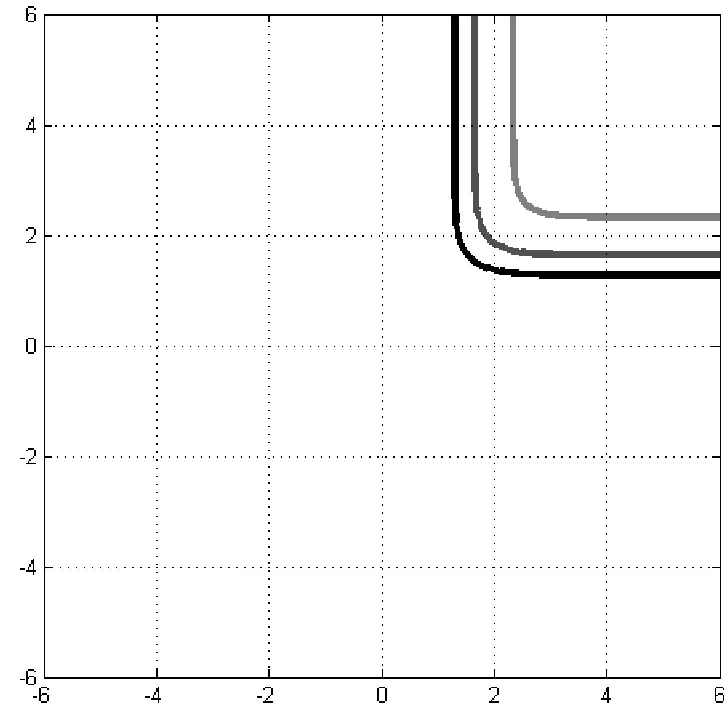
But it holds over a bounded box.

Illustration

Two-dimenisonal standard normal distribution, covariance = 0.5



Contours of density function



Contours of distribution function

Consider eigenvalues of $-\nabla^2 \log F(\boldsymbol{z})$.

Larger eigenvalue: $\omega(\boldsymbol{z})$.

Smaller eigenvalue: $\alpha(\boldsymbol{z})$.

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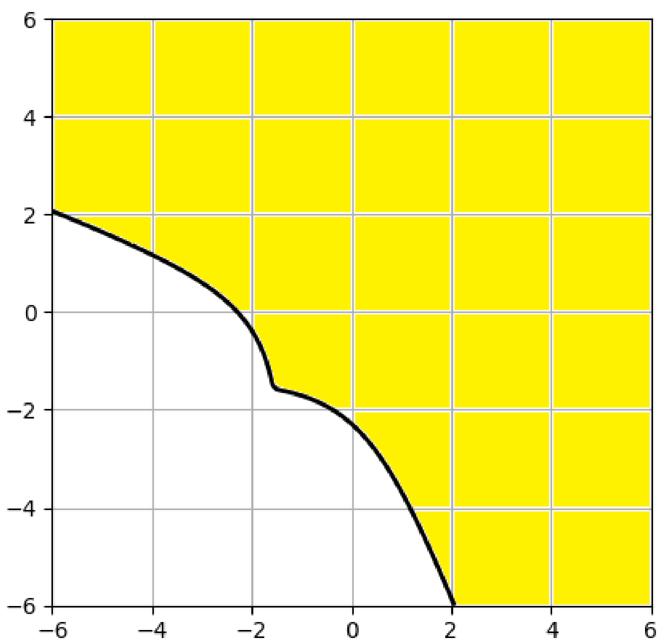
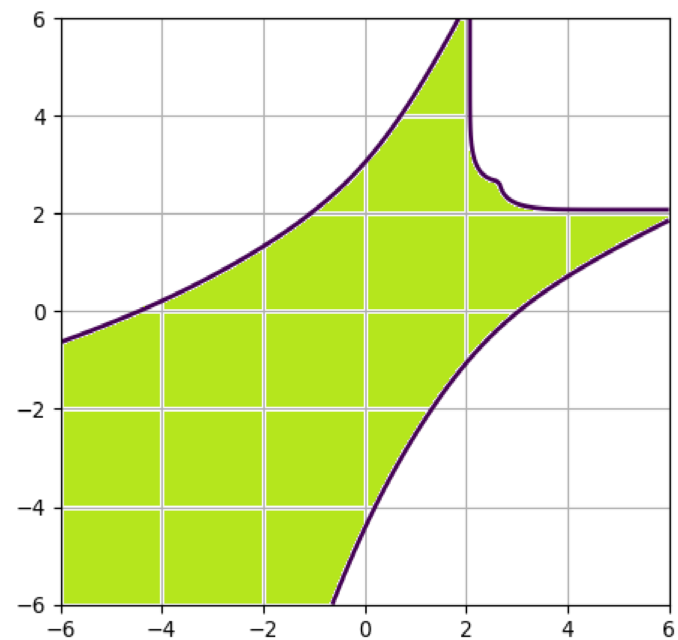
Level sets of eigenvalue functions



$\omega(\boldsymbol{z}) \le 10$



$\alpha(\boldsymbol{z}) \ge 0.1$

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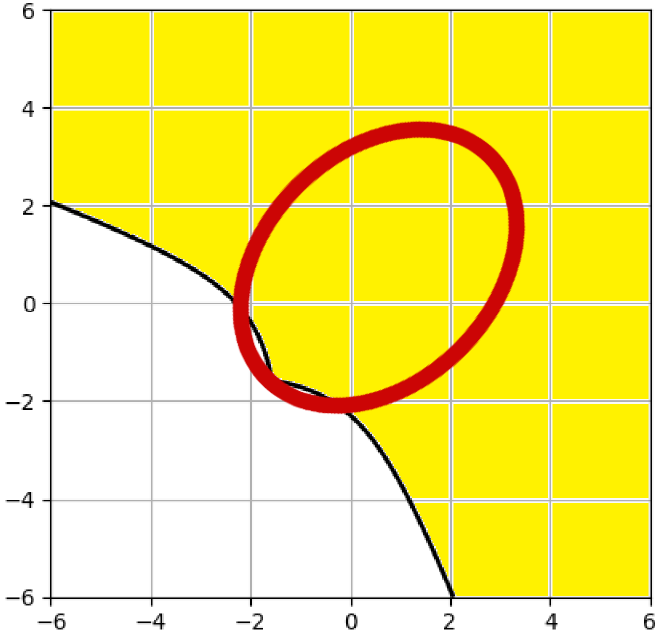

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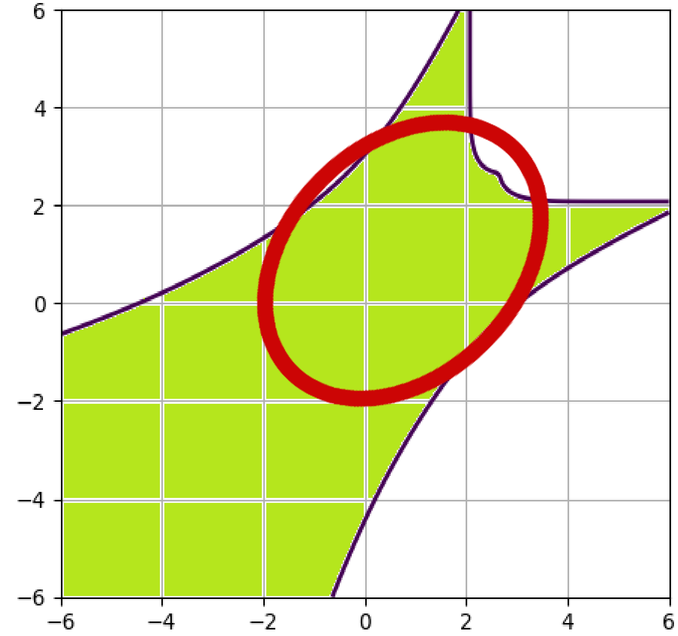

$\alpha(\boldsymbol{z}) \geq 0.1$

Working with gradient estimates

Feasibility study: comparison of direct cutting-plane method and dual approach

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Direct cutting-plane method:

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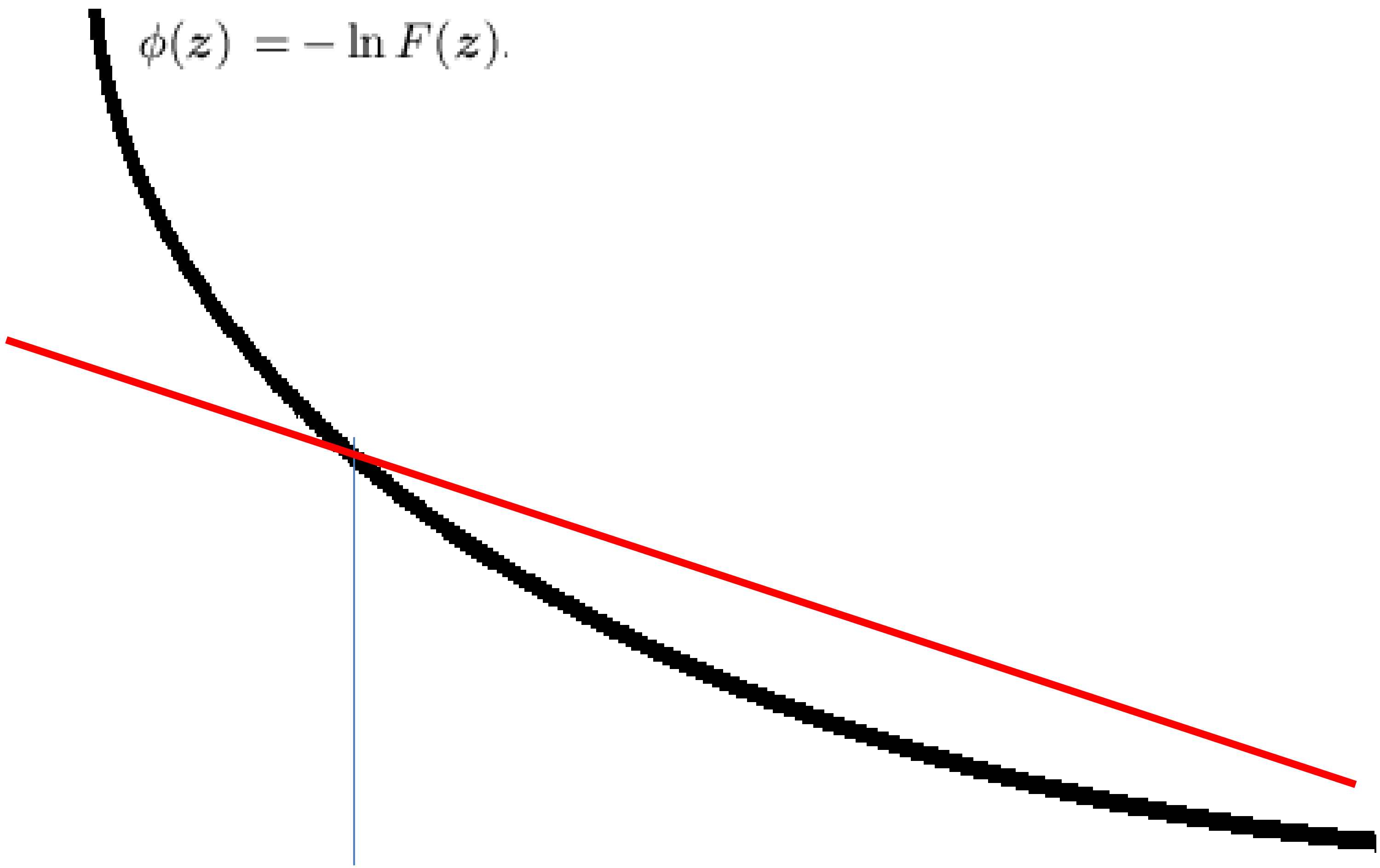
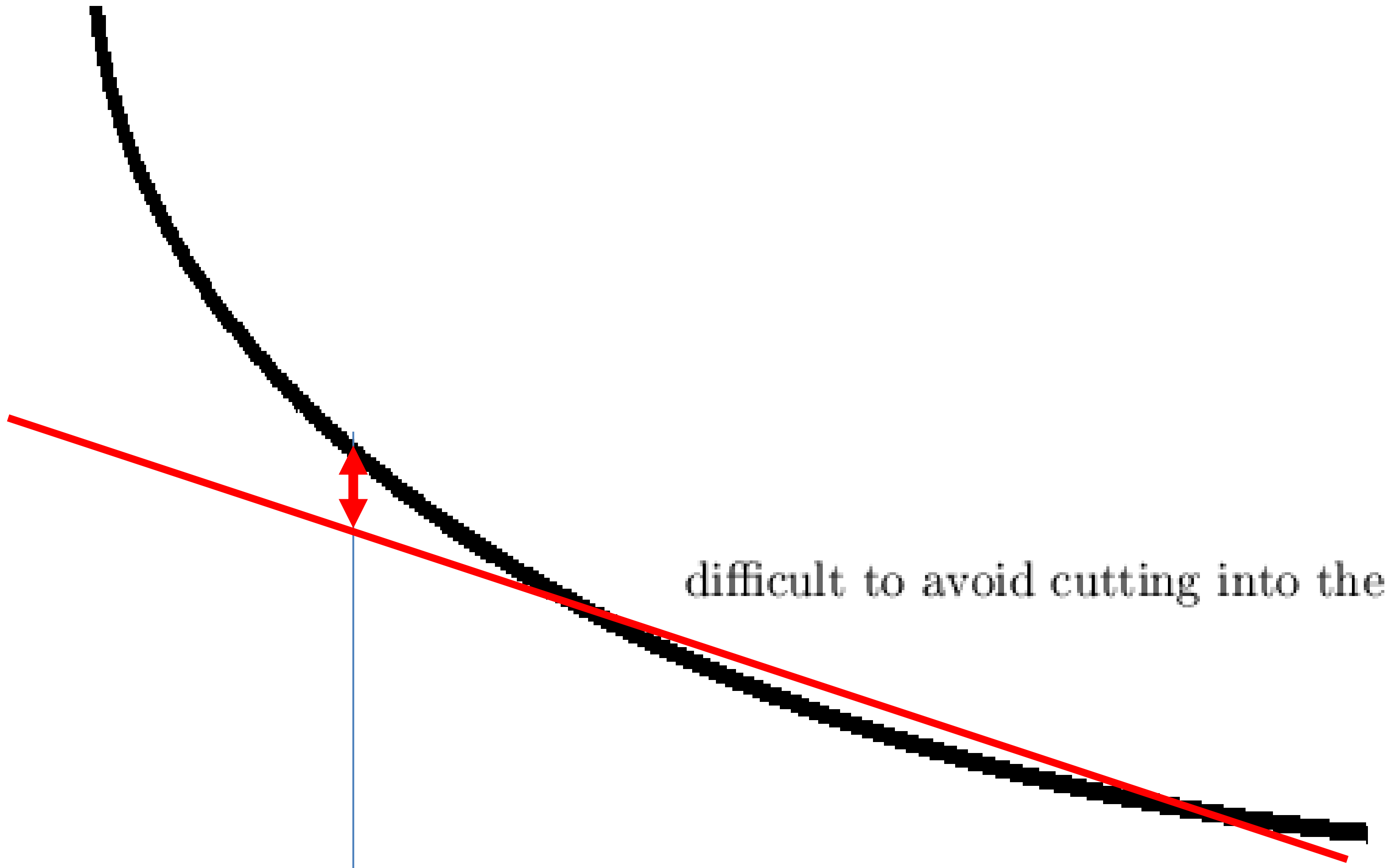


Working with gradient estimates

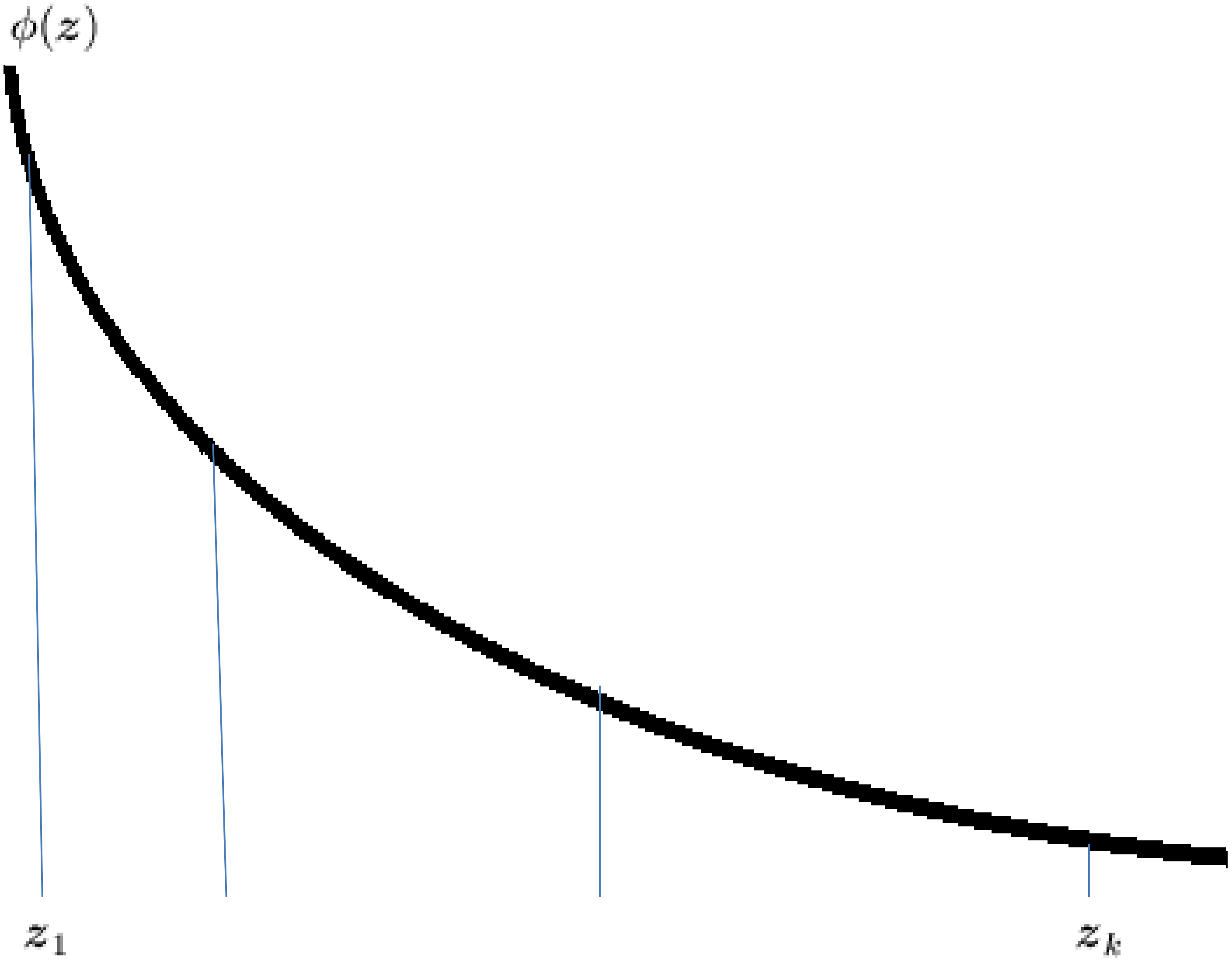
Feasibility study: comparison of direct cutting-plane method and dual approach

Direct cutting-plane method:



difficult to avoid cutting into the epigraph

Epi-approximation



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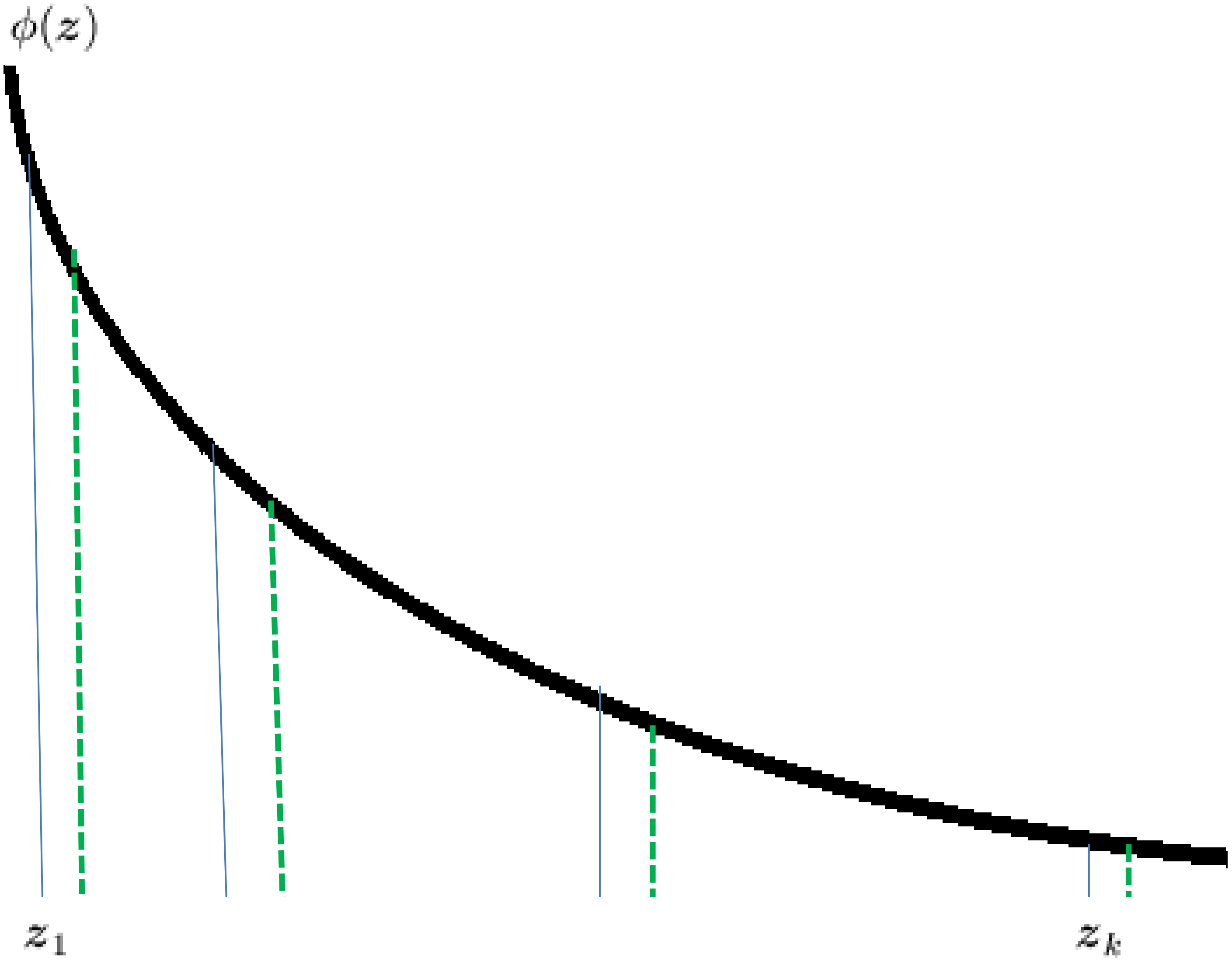

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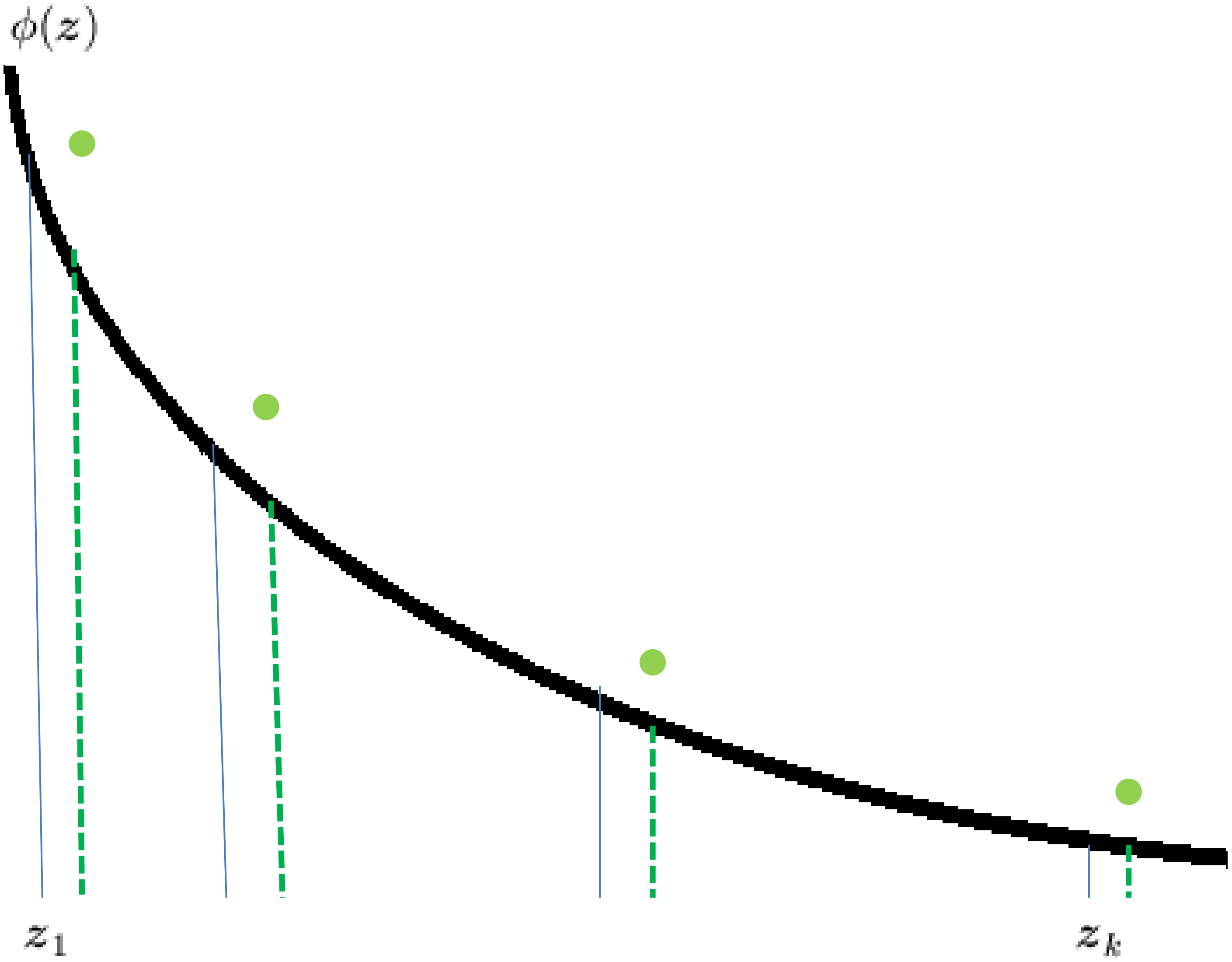

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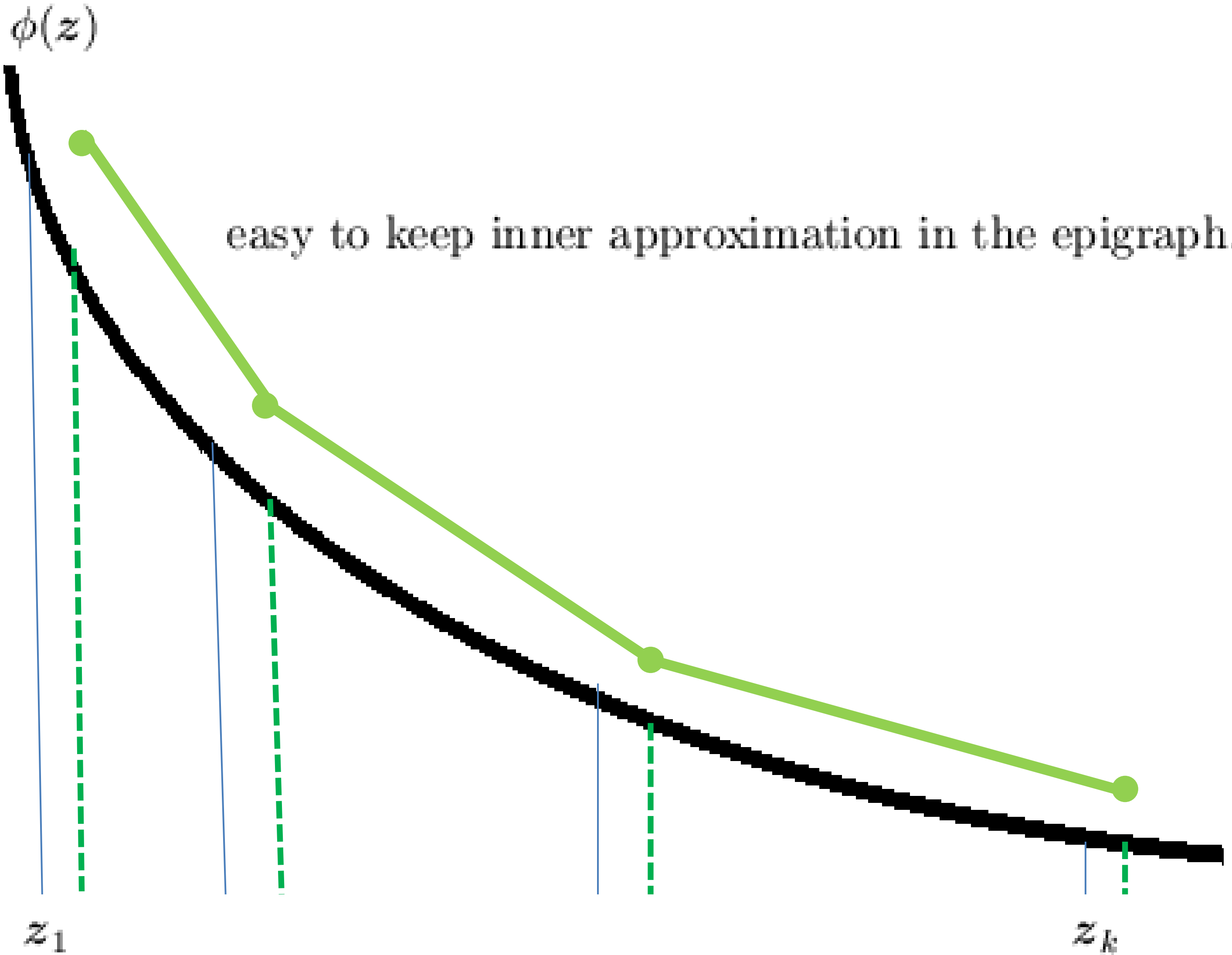

Working with gradient estimates

Theoretical development for an ideal convex programming problem

We wish to minimize $f(\boldsymbol{z})$ with a steepest descent method.

$$\alpha I \preceq \nabla^2 f(\boldsymbol{z}) \preceq \omega I \qquad (\boldsymbol{z} \in \mathbb{R}^n).$$

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Given $\sigma > 0$, we can construct realizations of a random vector $\boldsymbol{G}^\circ$ satisfying

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A generalization of the convergence theorem

Starting from $\boldsymbol{z}^0$, let $\boldsymbol{z}^1, \ldots, \boldsymbol{z}^j, \ldots$ denote the iterates.

Then we have

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Resemblance to the stochastic approximation family

But present approach builds a model problem.

Working with gradient estimates

Application to probability maximization

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Reliable gradient estimates can be constructed using ideas of

Szántai (1976, 1985);

Deák (1980, 1986);

Ambartzumian et al. (1998);

Gassmann (1988); Deák, Gassmann, Szántai (2002);

Mádi-Nagy, Prékopa (2004).

Köszönöm a figyelmet!