MTA SZTAKI
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Pareto-optimális anyagelosztó mechanizmus

XXXII. Magyar Operációkutatás Konferencia

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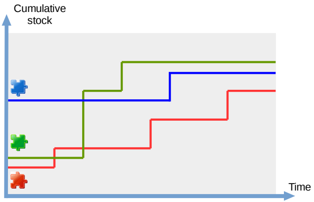
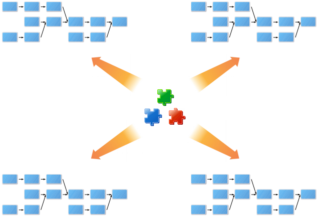
Outline

Introduction: project scheduling problem

- Projects with due dates
- Precedence relations between jobs
- Non-renewable resource constraints
  - Components arrive periodically
  - Example: production with scarce components (innovative components, long supply lead-times)
- Goal: minimize maximal tardiness
- Poly-time algorithm by Carlier and Rinnooy Kan[1]

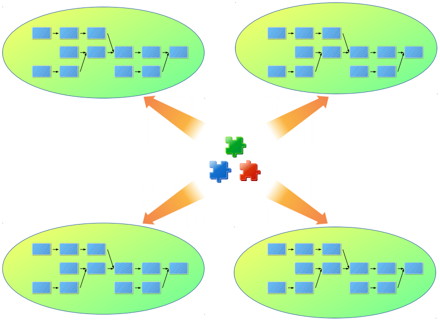




[1] Carlier and Rinnooy Kan: Scheduling subject to nonrenewable-resource constraints. Op. Res. Letters 1(2), 1982.

Introduction: mechanism design problem

- Conflicting goals
  - Global objective: minimize the maximal tardiness
  - The objective of each project agent is to minimize its tardiness
- Due dates of the projects are private information, all other information is public knowledge
- We seek a mechanism without money
- Related prior work: Course Allocation Problem



Assumptions:

- All parameters are deterministic
- Component demand is not greater than the supply
- Jobs start as soon as possible

Notation

- Parameters

h_p	due date of project p
$j \in J_p$	jobs of project p
$(j_1, j_2) \in A_p$	immediate precedence relations of project p
t_j	processing time (runtime) of job j
$d_{c,j} \geq 0$	demand of job j for component c
$a_{c,T_i} \geq 0 \quad i = 1, \dots, m$	number of components c arriving at time T_i

- Decision variables

$\mu_{c,j,T_i} \geq 0$	number of components c allocated to job j at time T_i
$\mu = \{ \mu_{c,j,T_i} \}$	allocation

- Derived variables

$$s_j^{(\mu)} = \min \left\{ s \,\middle|\, \forall c : \sum_{T_i \leq s} \mu_{c,j,T_i} \geq d_{c,j} \wedge \forall (j', j) \in A_p : e_{j'}^{(\mu)} \leq s \right\}$$

$$e_j^{(\mu)} = s_j^{(\mu)} + t_j$$

$$F_p^{(\mu)} = \max \left(\max_{j \in J_p} e_j^{(\mu)} - h_p, 0 \right)$$

Properties

Definition (Truthfulness (informal))

A mechanism is truthful if the projects always find it best to declare their true due dates.

Definition (Preference)

Project p prefers allocation μ' to μ ($\mu' \succ_{p,h_p} \mu$), if $F_{p,h_p}^{(\mu')} < F_{p,h_p}^{(\mu)}$.

Project p weakly prefers allocation μ' to μ ($\mu' \succeq_{p,h_p} \mu$), if $F_{p,h_p}^{(\mu')} \leq F_{p,h_p}^{(\mu)}$.

Definition (Pareto-optimality)

An allocation μ is Pareto-optimal if $\nexists \mu' : \forall p \in P : \mu' \succeq_{p,h_p} \mu$ and $\exists p \in P : \mu' \succ_{p,h_p} \mu$.

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Remark

Not every optimal solution is Pareto-optimal, but there is at least one.

The naïve mechanism

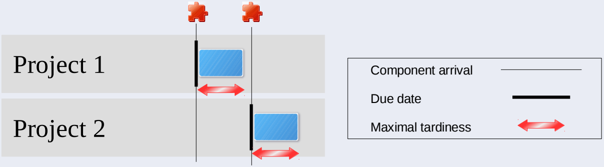
1. The projects announce their due dates to the central inventory
2. The inventory computes optimal schedule with the algorithm of Carlier and Rinnooy Kan
3. The inventory allocates the materials according to the schedule

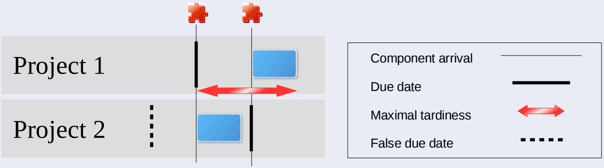
Truthfulness

Theorem

The naïve mechanism is not truthful.

Proof.





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Serial Dictatorship Mechanism

1. Let $p_1, \ldots, p_n$ be an arbitrary ordering of the projects
2. The projects announce their due dates to the central inventory
3. Inventory repeats from $k = 1$ to n
   3.1 Compute allocation $\mu^{(k)}$ such that
      - Projects $p_1, \ldots, p_{k-1}$ weakly prefer $\mu^{(k)}$ to $\mu^{(k-1)}$
      - Tardiness of p_k is minimal
      - Tardiness of projects $p_{k+1}, \ldots, p_n$ is disregarded
4. The inventory allocates the materials according to the schedule $\mu^{(n)}$

Carlier – Rinnooy Kan algorithm (sketch)

- Let $U(j)$ be the set of all successors of job j, and W_{jk} be the weight of the maximal path length between jobs j and $k \in U(j)$
- $f_j(t) = \max\{\, t - h_p, 0 \,\}$ if $j \in J_p$
- $\gamma_{ij} = \max \begin{cases} f_j(T_i + t_j) \\ \max\{\, f_k(T_i + t_k + W_{jk}) \mid k \in U(j) \,\} \end{cases}$
- $B_{ic} = \sum_{\tau=1}^{i} a_{c,T_\tau}$
  the amount of component c arriving until time T_i
- We seek the smallest γ such that
  $\forall c, i : \sum\{\, d_{c,j} \mid \gamma < \gamma_{ij} \,\} < B_{ic}$
- For a fixed i the smallest γ_i^* can be found with a median finding procedure, and the optimal $\gamma^* = \max_i \gamma_i^*$

Computation of SDM

- In step k we use the following $\gamma_{ij}^{(k)}$ instead of γ_{ij}:

$$\gamma_{ij}^{(k)} = \begin{cases} \gamma_{ij}, & j \in J_{p_k} \\ \infty, & j \in J_{p_{k'}} \wedge k' < k \wedge \gamma_{ij} > F_{p_{k'}, h_{p_{k'}}}^{(\mu^{k-1})} \\ 0 & \text{otherwise} \end{cases}$$

- We seek the smallest γ such that
  $\forall c, i : \sum \{ d_{c,j} \,|\, \gamma < \gamma_{ij}^{(k)} \} < B_{ic}$

Positive results

Theorem (Truthfulness)

The Serial Dictatorship mechanism is truthful.

Proof (sketch).

- Steps $1, \ldots, k-1$: h_{p_k} is disregarded
- Step k: tardiness of project k is minimized
- Steps $k+1, \ldots, n$: tardiness of project k remains constant □

Theorem (Pareto optimality)

The Serial Dictatorship mechanism is Pareto optimal.

Proof (sketch).

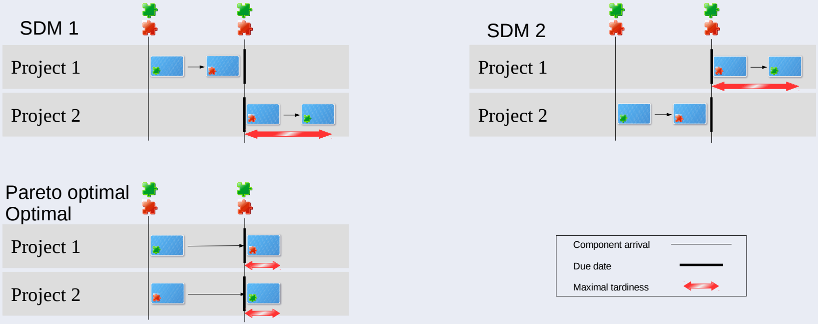
Let's indirectly assume $\exists \mu', p_k : \forall p \in P : \mu' \succeq_{p,h_p} \mu$ and $\mu' \succ_{p_k,h_{p_k}} \mu$. This contradicts optimality in step k. □

Negative results #1

Theorem

Not every Pareto optimal solution can be generated by a SDM.

Proof.



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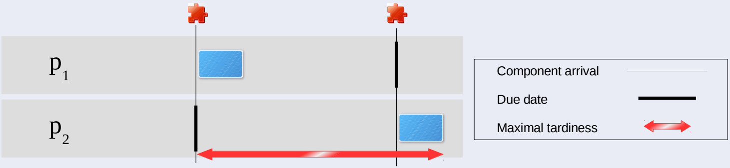
Negative results #2

Theorem

The resulted maximal tardiness of the SDM can be arbitrary far from the optimal one.

Proof.



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Numerical study

- Number of jobs per project: 5
- Processing times $\sim U(1,5)$
- Due dates $\sim U(1,50)$
- Demands for components $\sim U(0,5)$
- Graph density: 0.2
- Number of problem instances: 1000

- The average error decreases with
  - More frequent supply
  - Less components
- The maximum error decreases with
  - More projects

Average error

10 projects		Components					
		1	2	4	6	8	10
Supplies	5	16%	25%	42%	40%	45%	49%
	10	7%	8%	11%	15%	13%	17%
	15	2%	3%	4%	5%	7%	8%

50 projects		Components					
		1	2	4	6	8	10
Supplies	5	15%	23%	37%	48%	57%	58%
	10	3%	7%	11%	15%	17%	17%
	15	1%	2%	4%	5%	6%	8%

100 projects		Components					
		1	2	4	6	8	10
Supplies	5	16%	26%	38%	46%	58%	57%
	10	4%	6%	10%	13%	15%	20%
	15	1%	2%	4%	5%	7%	8%

Maximal error

10 projects		Components					
		1	2	4	6	8	10
Supplies	5	700%	600%	2900%	500%	2300%	1300%
	10	625%	700%	400%	500%	600%	540%
	15	250%	320%	200%	440%	300%	333%

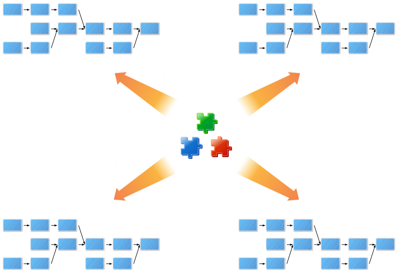
50 projects		Components					
		1	2	4	6	8	10
Supplies	5	420%	550%	420%	550%	363%	300%
	10	144%	125%	178%	171%	200%	188%
	15	70%	89%	100%	70%	71%	89%

100 projects		Components					
		1	2	4	6	8	10
Supplies	5	267%	350%	343%	282%	333%	310%
	10	150%	338%	156%	167%	175%	188%
	15	90%	100%	80%	70%	73%	111%

Summary and future work

- Serial Dictatorship Mechanism for material allocation
  - Truthful
  - Pareto-optimal
  - Poly-time
  - Arbitrary ordering (e.g., randomized, based on project value)
  - But: arbitrary large error
- Open questions
  - Existence of truthful mechanism other than SDM?
  - Existence of truthful mechanism that generates all Pareto optimal solutions?
- Further analysis of more realistic problems
  - Fixed order quantity or fixed time period ordering policies
  - Similar projects with different components (features)

Thank you for your attention!



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