The ternary Goldbach problem

(Március 28, kedd, 14 óra, MTA Rényi Intézet nagyterem)

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Abstract

The ternary Goldbach conjecture (1742) asserts that every odd number greater than 5 can be written as the sum of three prime numbers. Following the pioneering work of Hardy and Littlewood, Vinogradov proved (1937) that every odd number larger than a constant C satisfies the conjecture. In the years since then, there has been a succession of results reducing C, but only to levels much too high for a verification by computer up to C to be possible ($C > 10^{1300}$). I proved the conjecture for all odd numbers greater than 10^{27}; a simple computer verification then suffices to show that it is valid for all odd numbers up to 10^{27}.

We will go over the main ideas in the proof, focusing on what happens on the major arcs.

The ternary Goldbach problem revisited, I and II

(Március 29 és 30, szerda–csütörtök, 14 óra, MTA Rényi Intézet nagyterem)

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Abstract

We will go over the technical heart of the proof, namely, minor-arc estimates.

The main point is to give bounds for exponential sums of the form $S(\alpha, x) = \sum_p e(\alpha p)\eta(p/x)$, where η is a smooth weight and α is an element $[0, 1]$ close to a/q. We want an elementary estimate that goes to 0 rapidly as q goes to infinity. It is essential for our purposes that our bound be "log-free", i.e., that $S(\alpha, x)/x$ be bounded in terms of q alone. Of course, the constants involved should be as small as possible.

The spirit of the proof goes in some sense back to Vinogradov. However, it is necessary to work from scratch, incorporating later ideas familiar from the study of other problems in analytic number theory. Vaughan's identity works as a gambit, where every log that is given away must be won back – each in a different way.

The close natural relation between the large sieve and the circle method comes from the fore. Small sieves are avoided, but then one such sieve appears naturally thanks to smoothing: the Barban–Vehov–Graham "natural" variant on the Selberg sieve, reworked so as to give explicit results.